Simulation Games

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Joint work with Krish Chatterjee and Jan Otop.

GRAPH GAMES IN FORMAL METHODS

1. Reactive synthesis (Church's problem)
2. Agent verification (ATL etc.)
3. Simulation relations

Reactive Verification:

given a stateful system implementation
(e.g., an automaton A)

and a stateful system specification
(e.g., another automaton B),

does the implementation conform to the specification?

Language inclusion: L(A) · L(B)   PSPACE

Reactive Verification:

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1. Language inclusion: L(A) · L(B) PSPACE

LINEAR TIME

2. Simulation relation [Milner] quadratic

BRANCHING TIME

Reactive Verification:

given a stateful system implementation
(e.g., an automaton A)

and a stateful system specification
(e.g., another automaton B),

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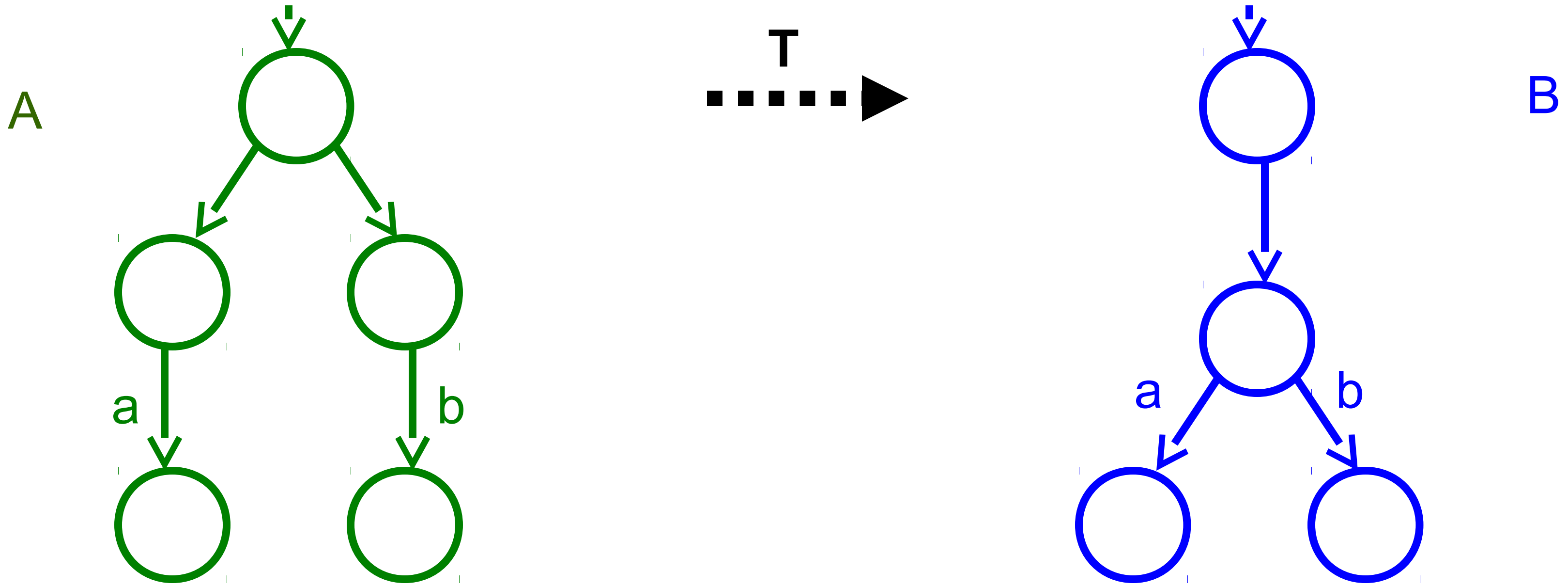
1. Language inclusion game on product — PSPACE
PARTIAL INFORMATION

2. Simulation game on product — quadratic
COMPLETE INFORMATION

SAFETY

Simulation

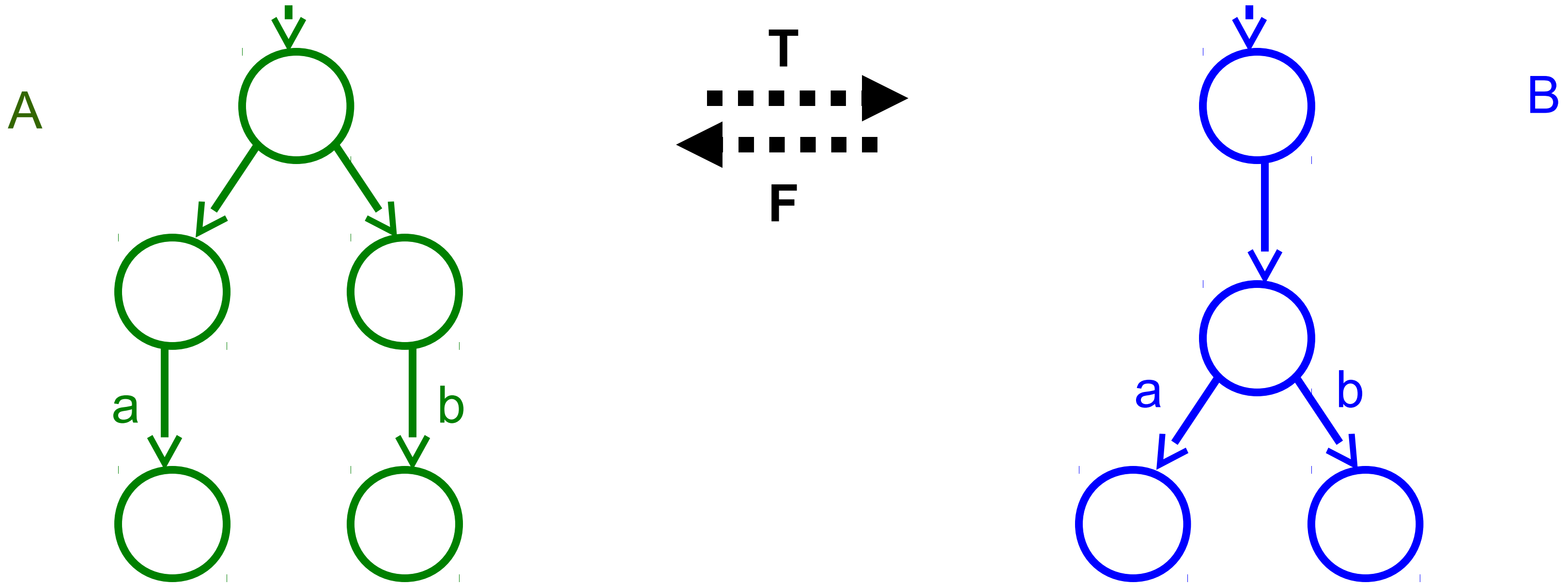


Challenger ▪▪▪▪▪▶ Simulator

T if Simulator wins
F if Challenger wins

SAFETY

Simulation



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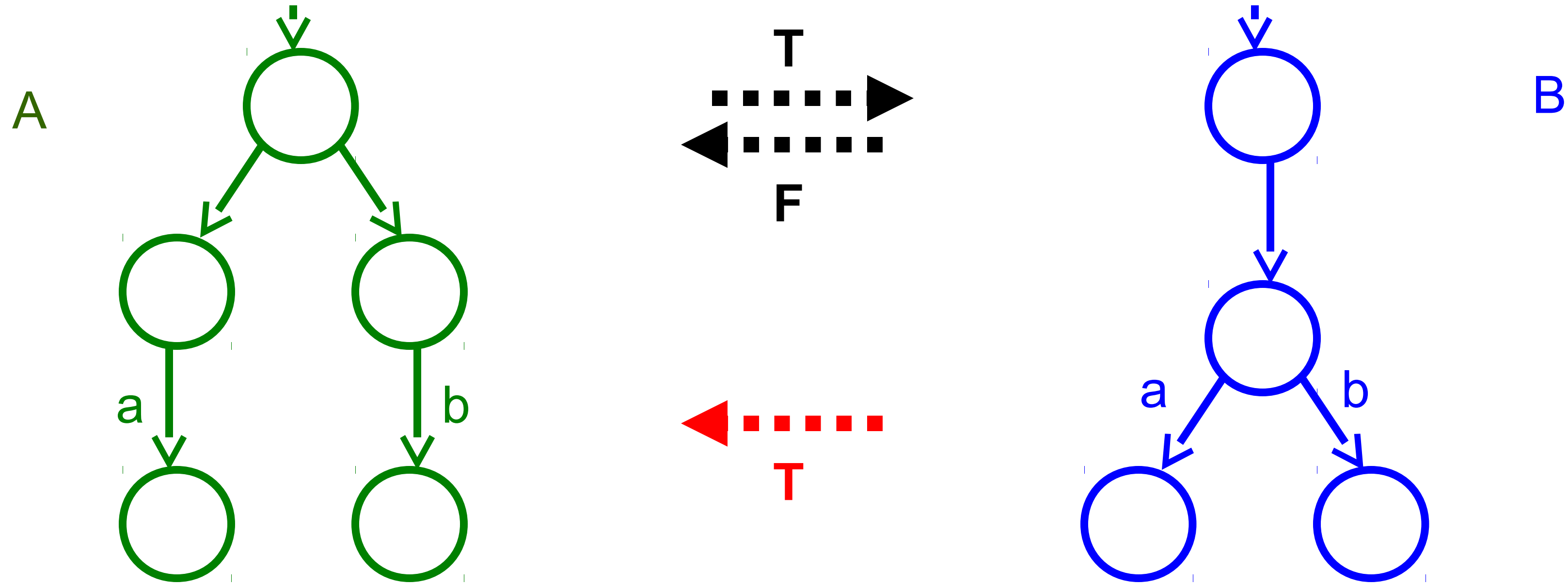

Language inclusion

Challenger ▪▪▪▪▪▶ Simulator

T if Challenger does not win

F if Challenger wins

SAFETY

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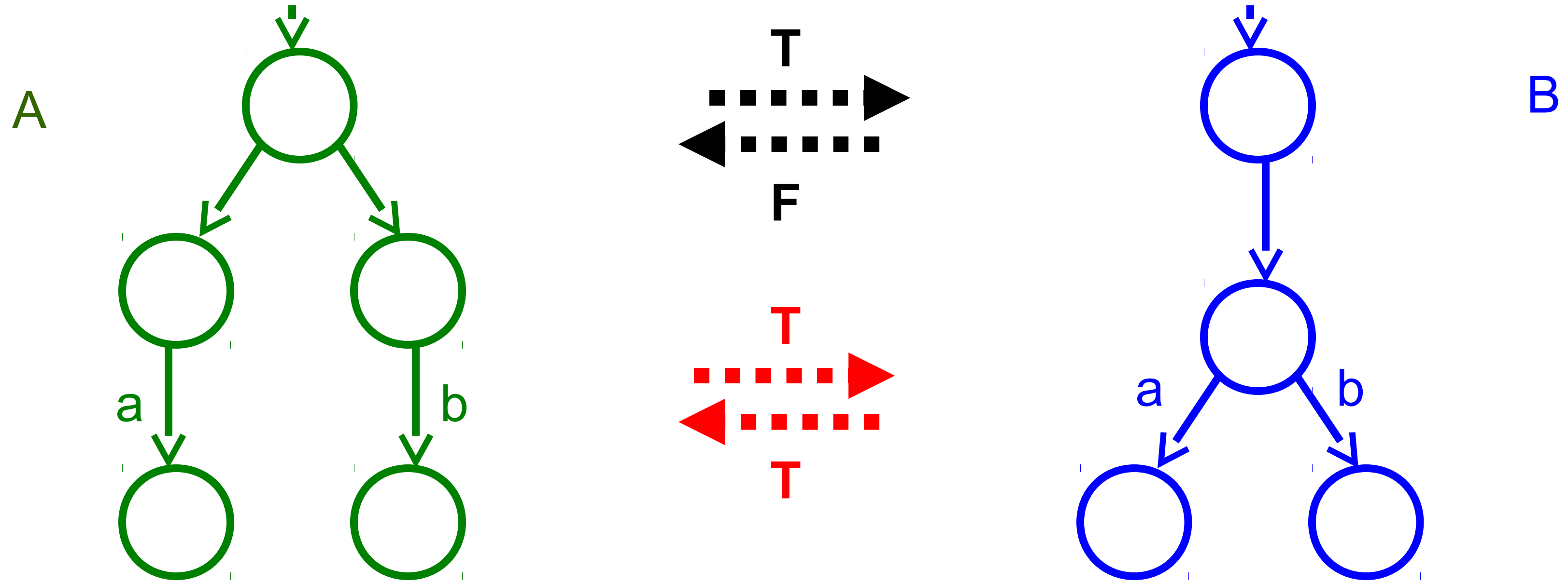


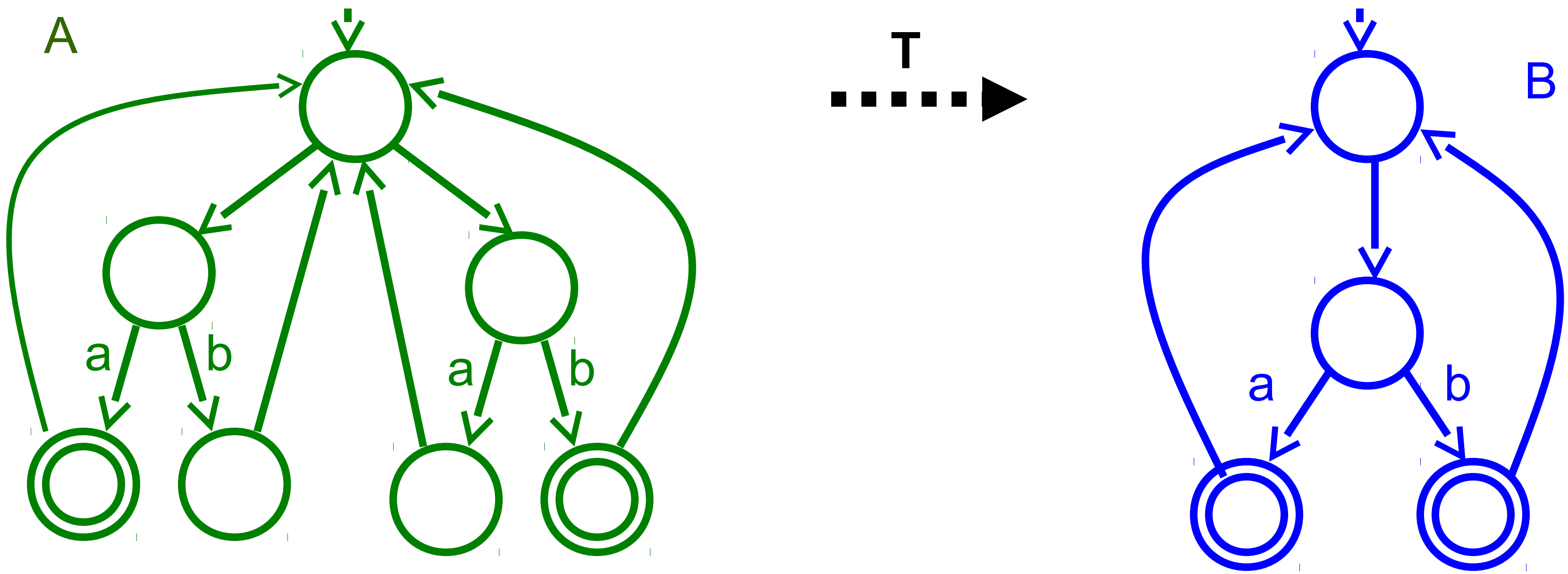
Language inclusion

Challenger ▸ Simulator

T if Challenger does not win
F if Challenger wins

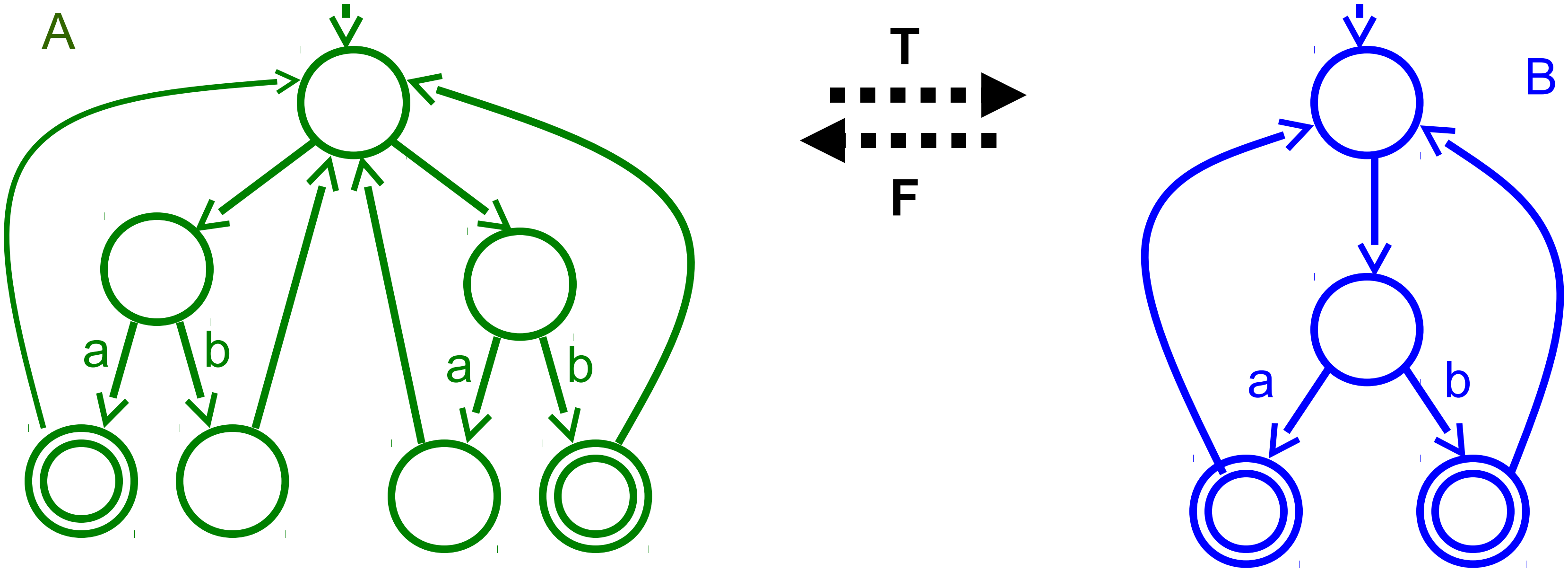
FAIRNESS

Simulation



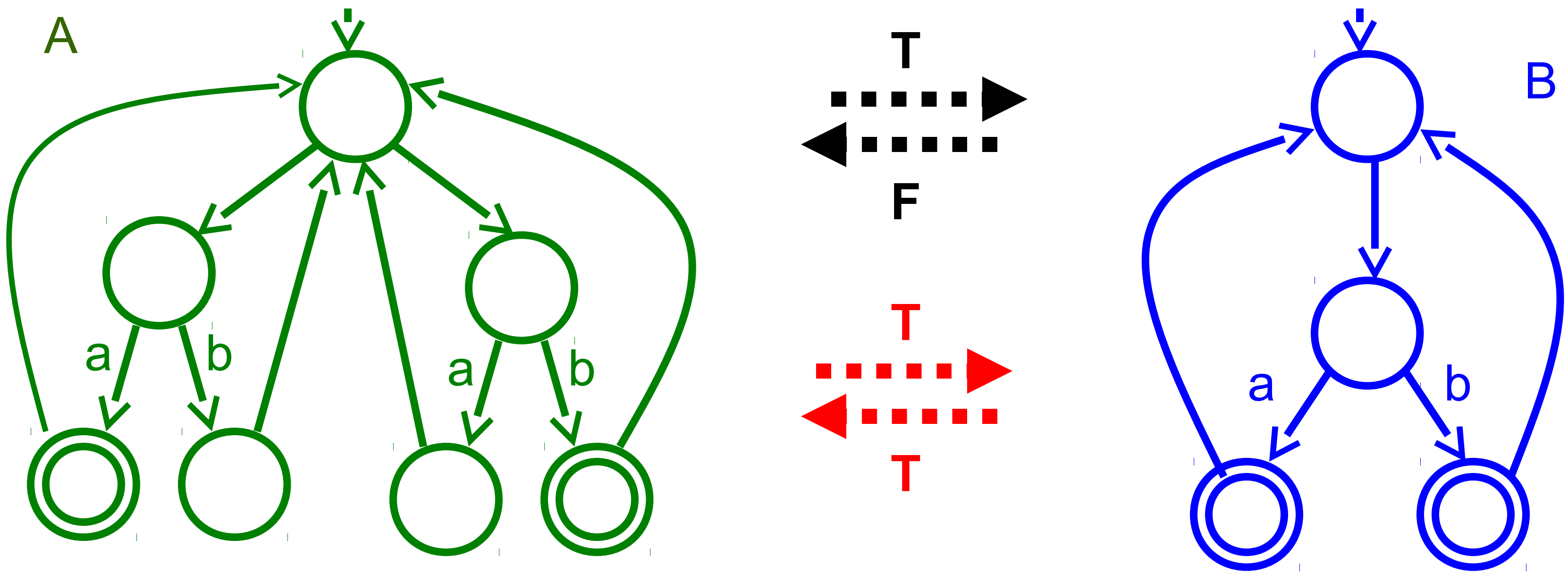
FAIRNESS

Simulation



FAIRNESS

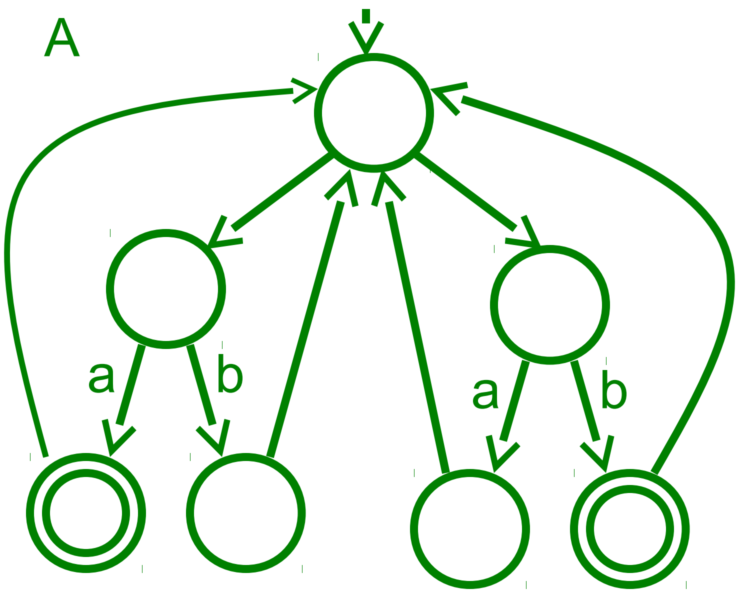
Simulation

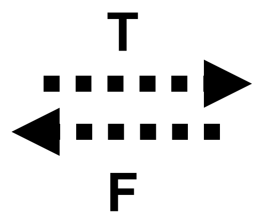


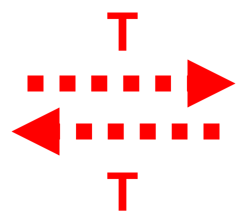
Language inclusion

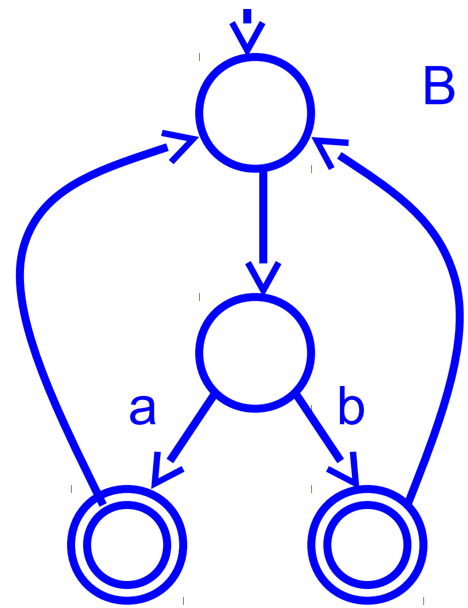
FAIRNESS

Simulation







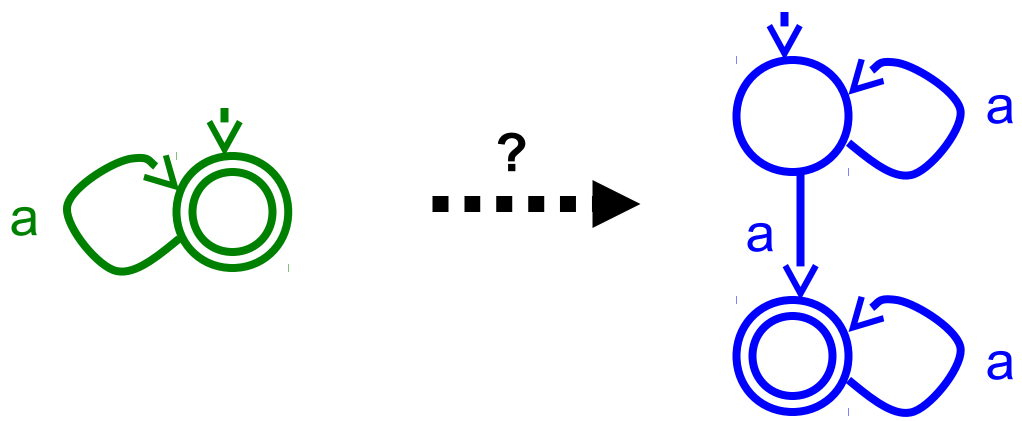


Language inclusion

Buchi automata !
1Streett game

FAIR SIMULATION

1. Lynch et al.: for every fair Challenger path, every matching Simulator path is fair
2. Grumberg et al.: FAIR-ACTL — for every fair Challenger path, there is a matching fair Simulator path
3. H/Kupferman/Rajamani CONCUR '97: game definition



SIMULATION GAMES

1. SAFETY: same as corresponding graph game
2. FAIRNESS: objective changes (Buchi ! 1Streett)
3. LIMSUP: solution complexity changes
4. LIMAVG: same
5. FAIR LIMAVG: strategy complexity changes
6. SUM: only simulation game exists
7. DISCOUNTED THRESHOLD: simulation game open

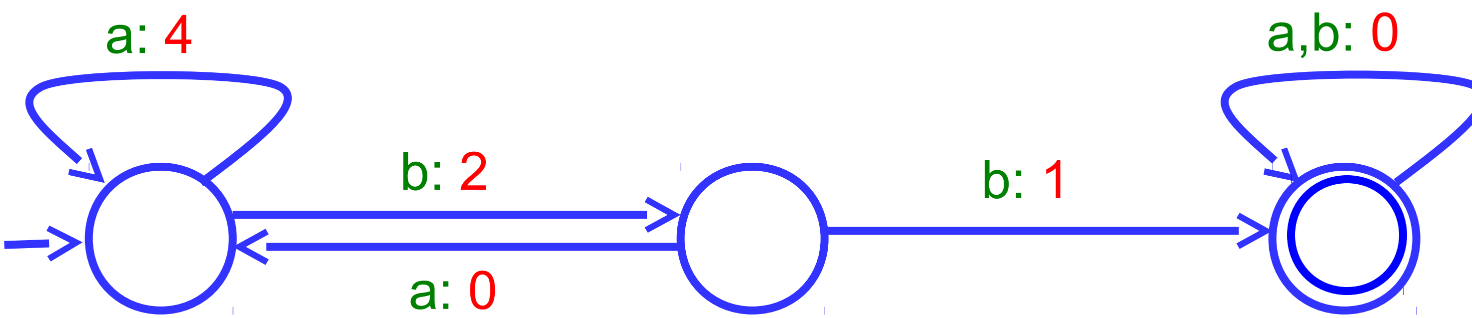
WEIGHTED AUTOMATA

Q: states

Σ: letters

q_0 in Q: initial state

δ µ Q × Σ £ R £ Q: transition function



path: $x = q_1 a_1 v_1 q_2 a_2 v_2 \ldots$

observation function: $obs(x) = a_1 a_2 a_3 \ldots$

value function: $val(x) = val(v_1 v_2 v_3 \ldots)$ 2 R

VALUE FUNCTIONS

Max value: $\mathrm{val}(v_1 v_2 v_3 \ldots) = \sup\{ v_i : i \geq 1 \}$

(only 0 and 1 weights: safety)

Limsup value: $\mathrm{val} = \lim_{n\to\infty} \sup\{ v_i : i \geq n \}$

(only 0 and 1 weights: Buchi)

Limavg value: $\mathrm{val} = \liminf_{n\to\infty} 1/n \cdot \sum_{1\leq i\leq n} v_i$

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Limavg value: $val = \liminf_{n \to \infty} 1/n \cdot \sum_{1 \leq i \leq n} v_i$

Energy value: $val = \inf\{ v : (\forall n \geq 1) (v \geq \sum_{1 \leq i \leq n} v_i) \}$

val(2,5,0,-3,-3,5,0,-3,-3,5,0,-3,-3,...) = 7

VALUE FUNCTIONS

Max value: $\mathrm{val}(v_1 v_2 v_3 \ldots) = \sup\{ v_i : i ¸ 1 \}$

(only 0 and 1 weights: finite condition)

Limsup value: $\mathrm{val} = \lim_{n!1} \sup\{ v_i : i ¸ n \}$

(only 0 and 1 weights: Buchi condition)

Limavg value: $\mathrm{val} = \mathrm{liminf}_{n!1}\ 1/n ¢ \sum_{1·i·n} v_i$

Energy value: $\mathrm{val} = \inf\{ v : (8\ n ¸ 1)\ (v ¸ \sum_{1·\ i·n} v_i) \}$

Discounted: $\mathrm{val} = \sum_{i¸\ 1} d^i ¢ v_i$ for some $0<d<1$

WEIGHTED AUTOMATA

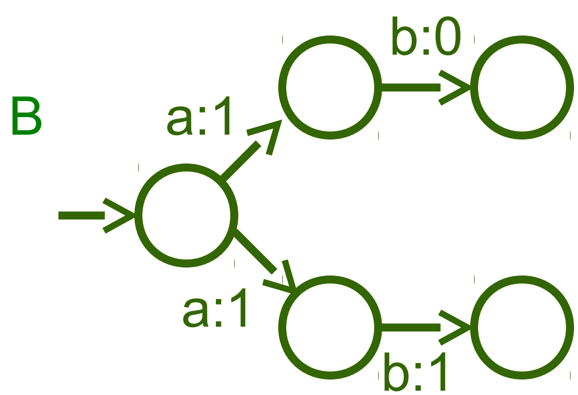
path $x = q_1 a_1 v_1 q_2 a_2 v_2 \ldots$

word $obs(x) = a_1 a_2 a_3 \ldots$ 2 Σ^ω

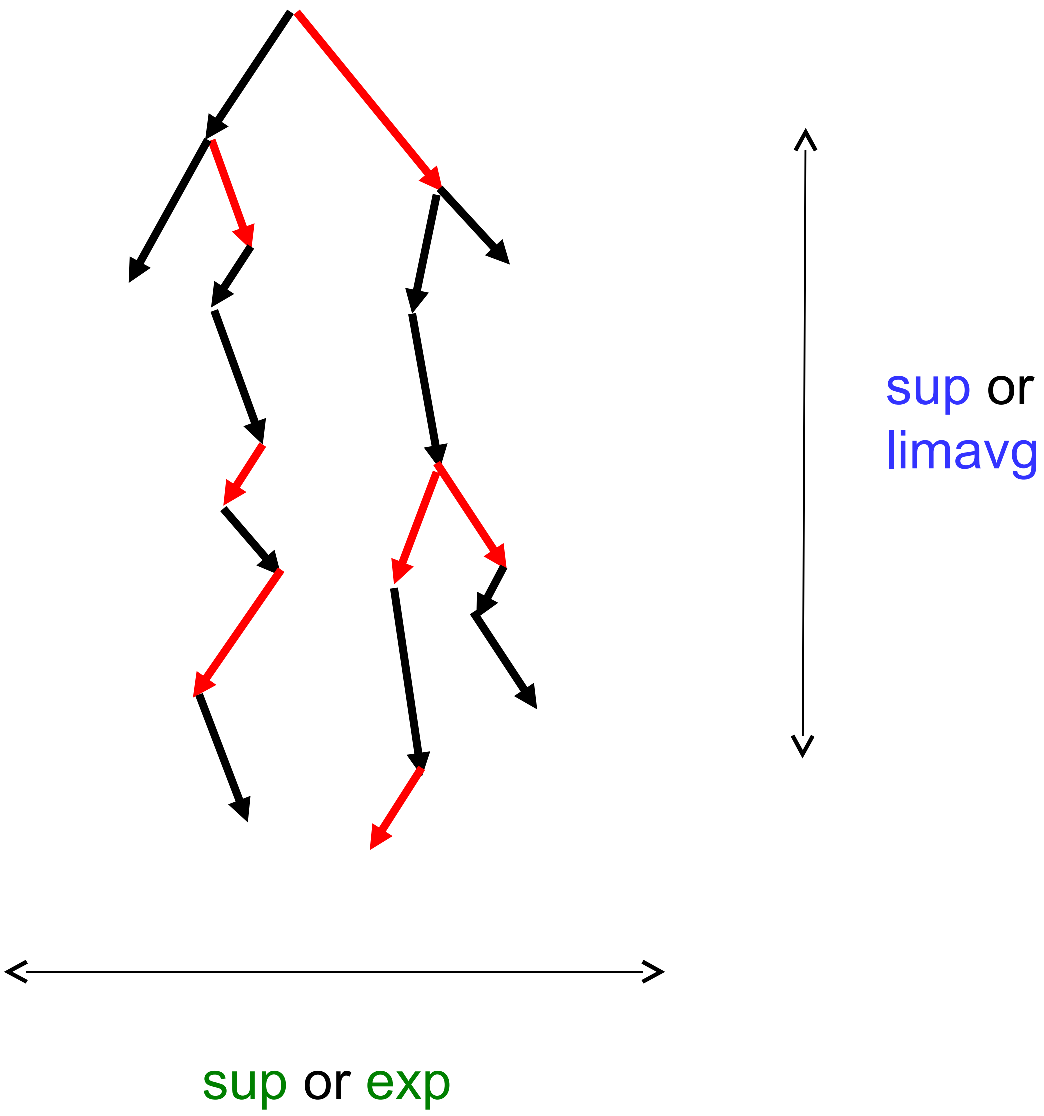
language   A: Σ^ω ! B   BOOLEAN

B: Σ^ω ! R   QUANTITATIVE

$B(w) = \sup_x \{ val(x) : obs(x) = w \}$



B(ab) = max(val(1,1), val(1,0))


sup or
limavg
sup or exp

QUANTITATIVE LANGUAGE INCLUSION

Given two weighted automata A and B, is L(A) · L(B)?

i.e. 8 w 2 Σ^ω : A(w) · B(w)

For finite and Buchi automata: PSPACE.
For max and limsup automata: PSPACE.
For limavg and energy automata: undecidable.
For discounted automata: open.

QUANTITATIVE SIMULATION GAMES

Given two weighted automata A and B,
B simulates A if in the simulation game on the product space,

if Challenger produces a (fair) path x of A
then Simulator produces a (fair) path y of B such that
obs(x) = obs(y) and val(x) · val(y).

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LIMSUP SIMULATION GAMES

Limsup graph game with weights 0,1,2,...k:
Player 1 maximizes the largest weight that is visited infinitely often;
Player 2 minimizes that weight.
Solution: k Buchi games 2 $O(kn^2)$.

Limsup simulation game with weights 0,1,2,...k:
Replace each Challenger weight v by 2v;
Replace each Simulator weight v by 2v+1.
Solution: parity game 2 NP Å coNP.

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LIMAVG SIMULATION GAMES

Limavg graph game:
Player 1 maximizes liminfavg = limsupavg;
Player 2 minimizes that value.
Memoryless strategies suffice, hence in NP Å coNP.

Limavg simulation game with weights:
Challenger maximizes limavg of Challenger weights c_i;
Simulator maximizes limavg of Simulator weights s_i.
Solution: limavg game on product graph with weights s_i-c_i.

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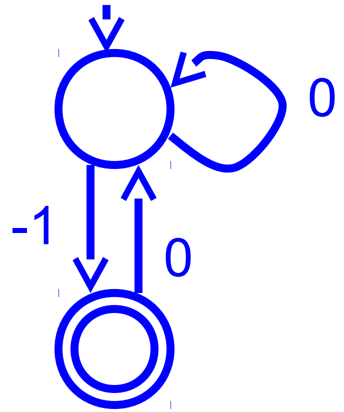
FAIR LIMAVG GRAPH GAMES

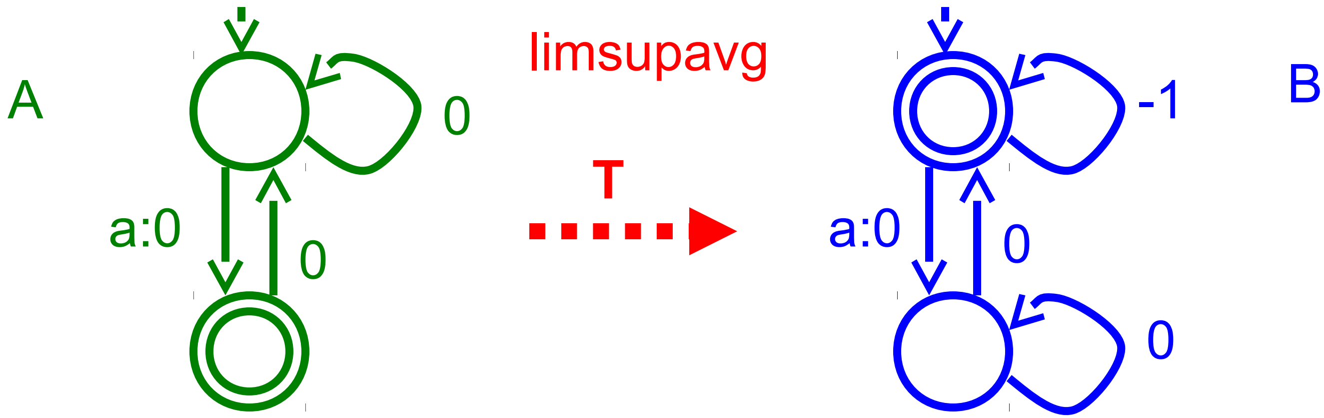
Memoryless strategies do not suffice:



Still, opponent has memoryless optimal strategy
and liminfavg = limsupavg.

FAIR LIMAVG SIMULATION GAMES

Challenger and Simulator need memory and liminfavg ≠ limsupavg:



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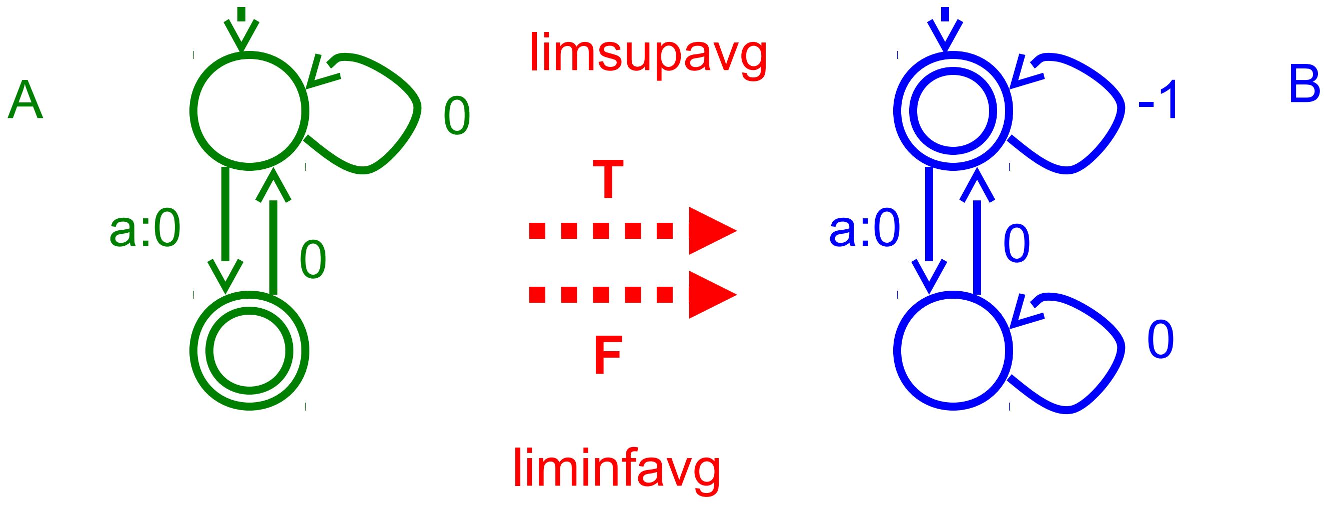

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SUM SIMULATION GAMES

There is no sum graph game with positive weights,
as the $\sum_{i \geq 1} v_i$ diverges.

But there is a sum simulation game:
Simulator wins if for all $n \geq 1$, $(\sum_{1 \leq i \leq n} c_i) \leq (\sum_{1 \leq i \leq n} s_i)$.
Solution: energy game on product graph with weights s_i-c_i.

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DISCOUNTED SIMULATION GAMES

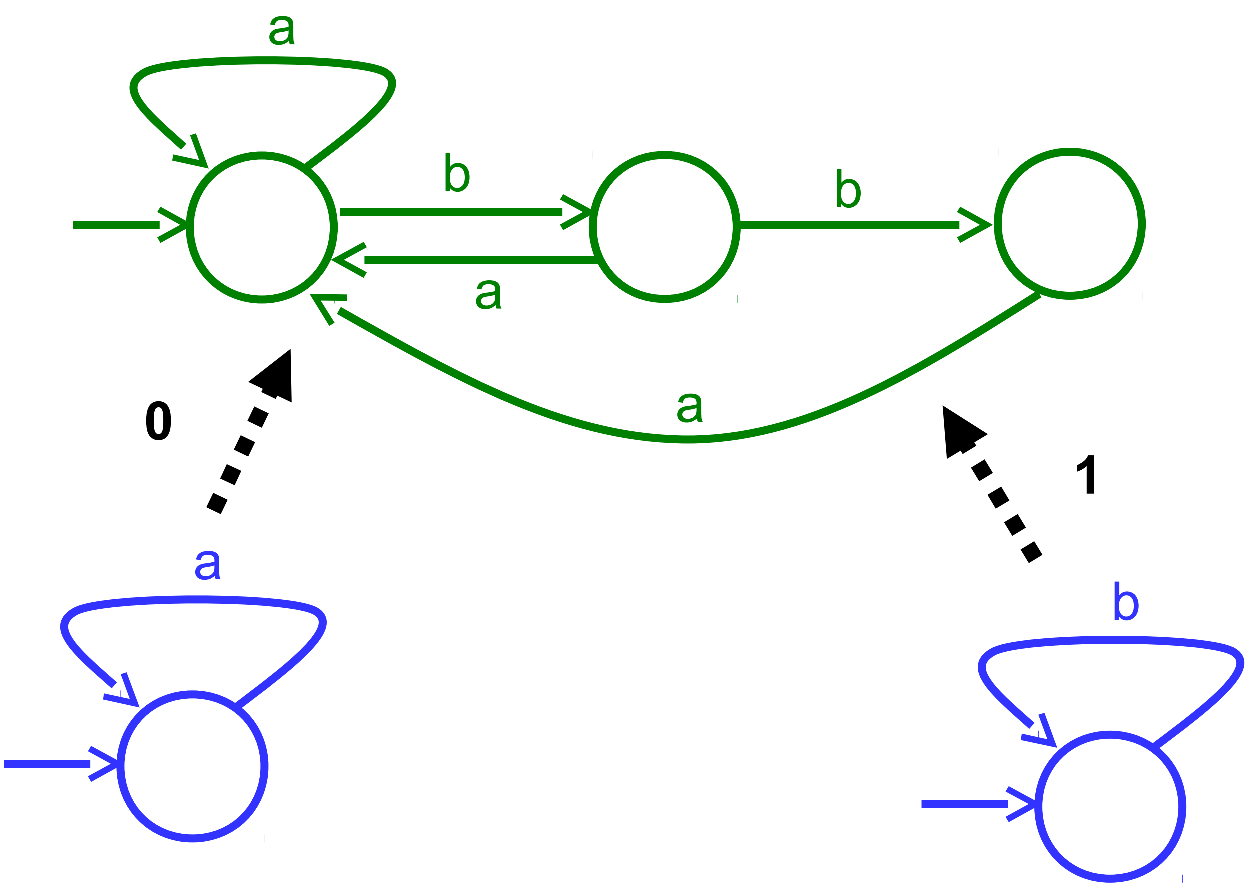
A discounted threshold graph game is won if $(\sum_{i ¸ 1} d^i v_i)$ ¸ T for a given discount factor d and threshold T.
In a threshold simulation game, the Simulator must beat the threshold if the Challenger does.

A discounted graph game or discounted threshold graph game can be solved, because memoryless strategies suffice.

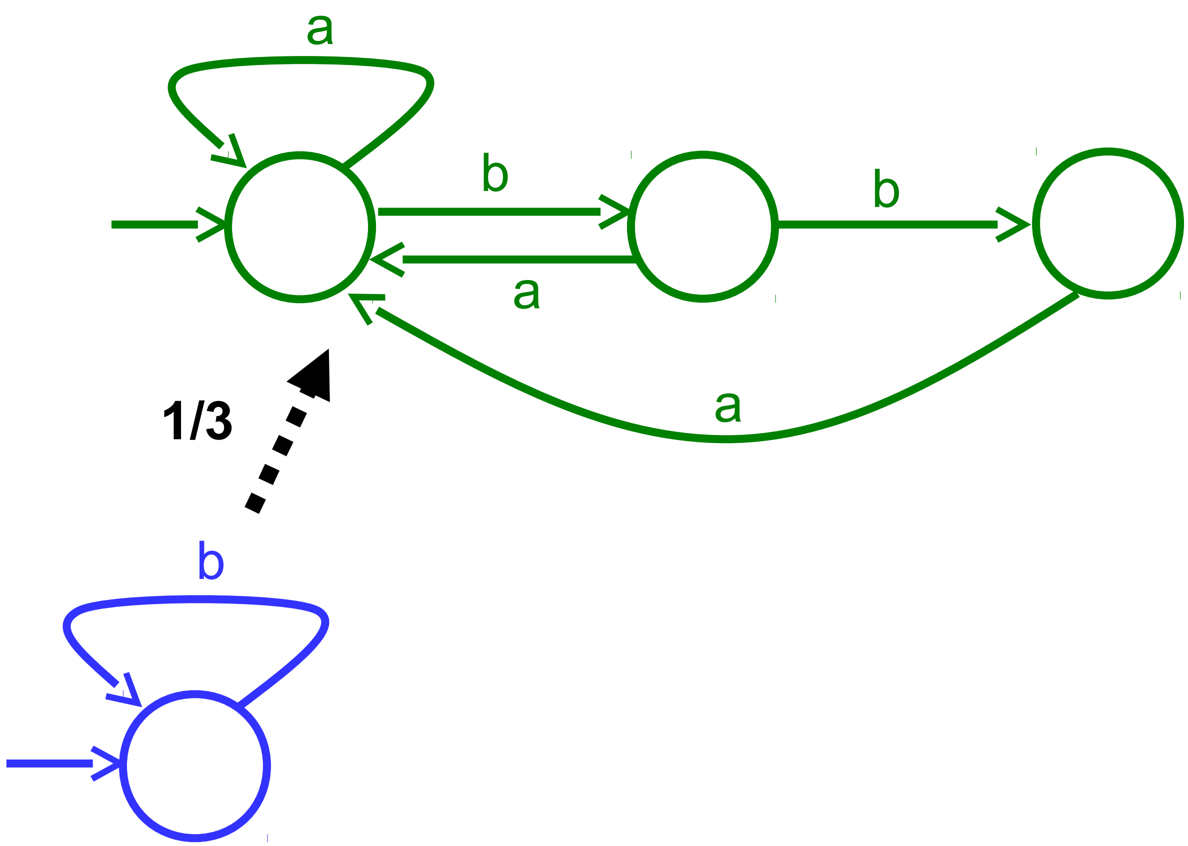
A discounted simulation game can be solved as a discounted game on the producr graph with weights c_i-s_i.

But discounted threshold simulation games are open.
Indeed the following basic question is open:
On a weighted graph, is there an infinite path with discounted value function exactly T?

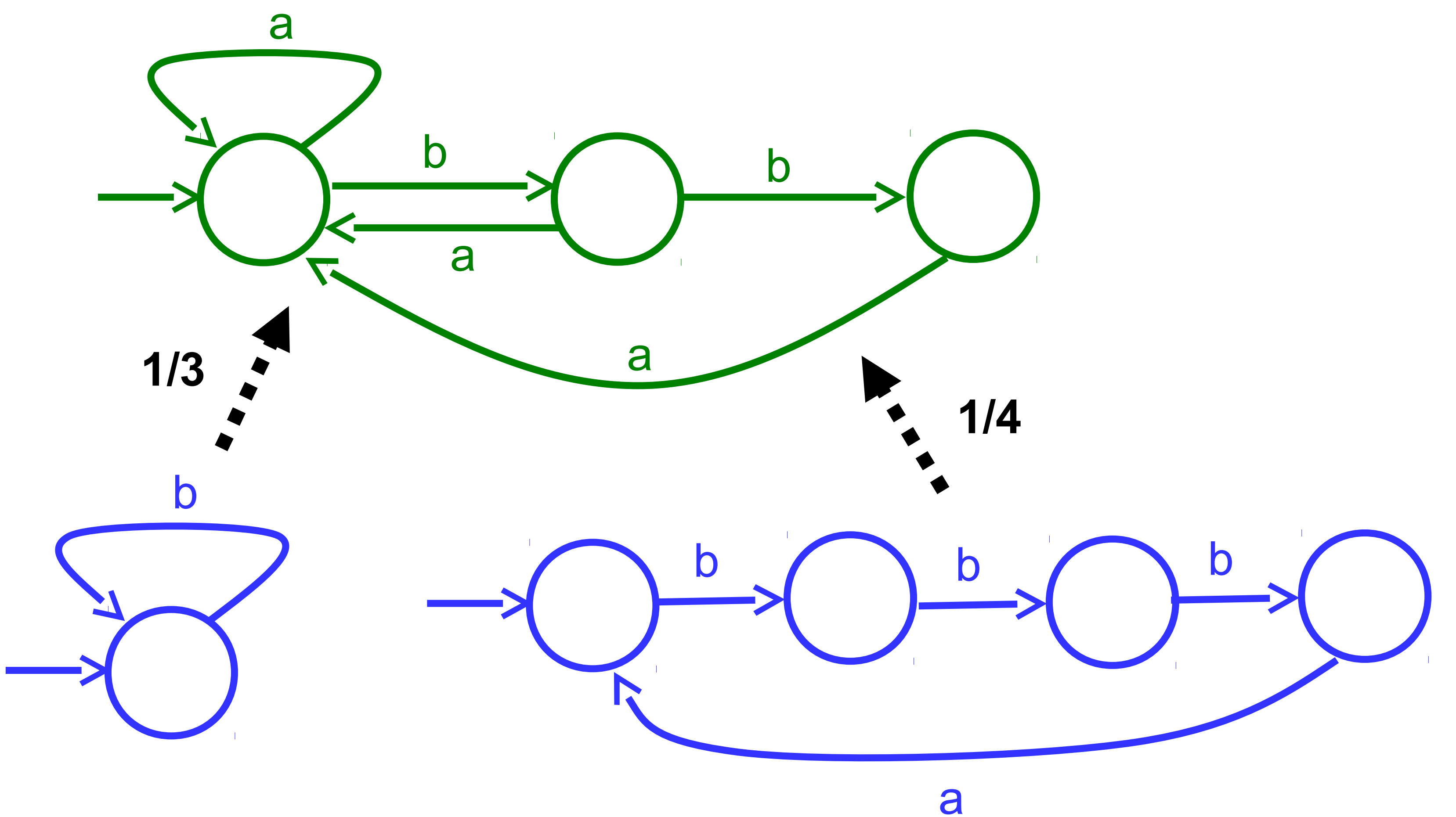
FROM SIMULATION GAMES



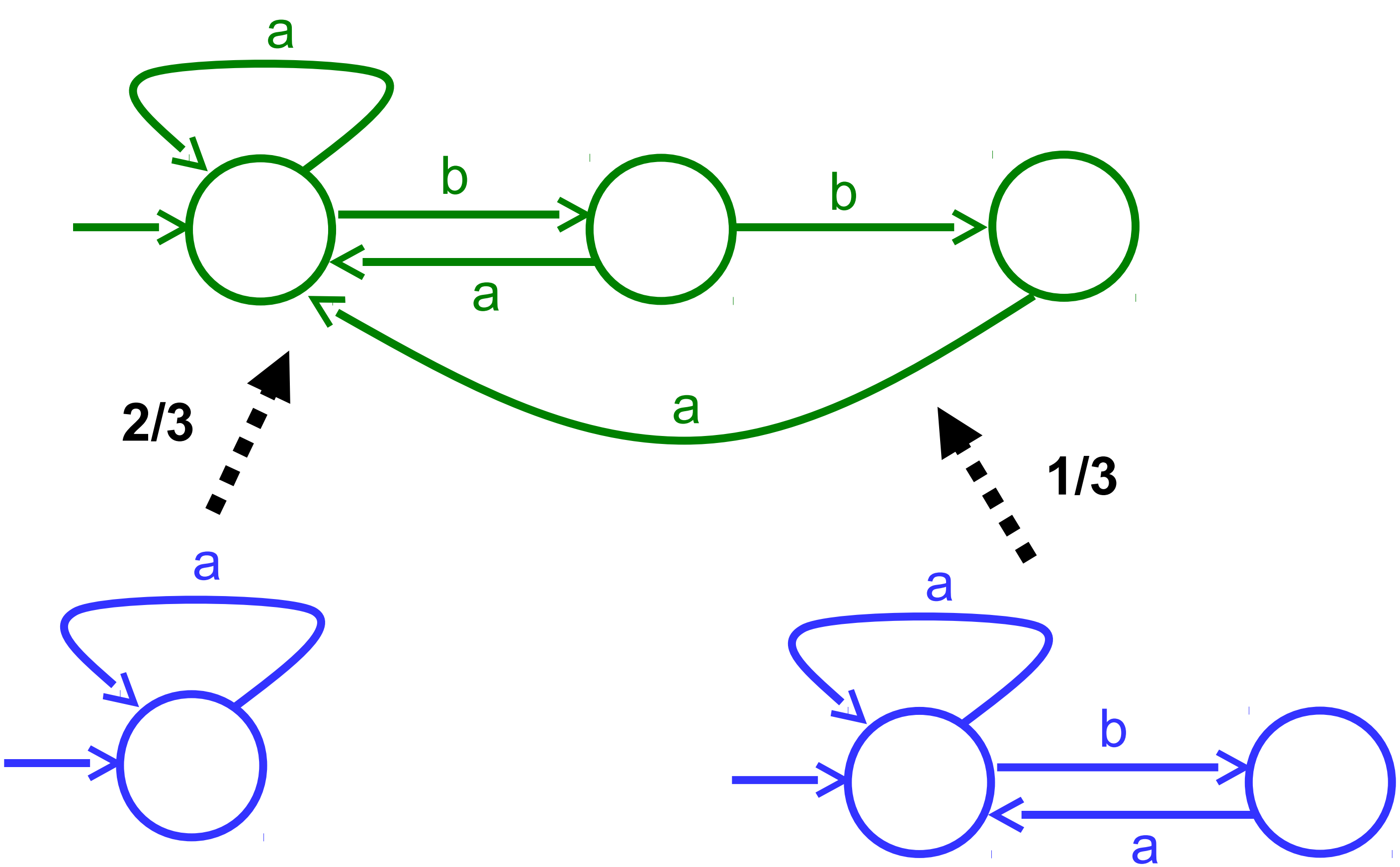
TO SIMULATION DISTANCES



CORRECTNESS DISTANCE



ROBUSTNESS DISTANCE



SOME REFERENCES

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Fair Limavg Games:
Chatterjee/H/Jurdzinski LICS ‘05

Simulation Distances:
Cerny/H/Radhakrishna CONCUR ‘10