Refining and Delegating Strategic Ability in ATL

Dimitar P. Guelev

IMI/BAS

http://www.math.bas.bg/~gelevdp

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Plan of Talk

Introduction: a converse to coalition abstraction in ATL

Preliminaries: ATL on Concurrent Game Models (CGMs)

Refinement of strategic abilities: the definition

- Γ-to-i homomorphisms of CGM
- the refinement modality $\langle i \sqsubseteq \Gamma \rangle \varphi$

Examples of refining strategic ability

Axioms and rules for the refinement modality

Using the refinement modality to model delegation

Model-checking the $\langle . \sqsubseteq . \rangle$-flat subset of $\mathrm{ATL}_{\sqsubseteq}$

- the $\langle\langle . \rangle\rangle \circ$-subset
- the whole of $\langle . \sqsubseteq . \rangle$-flat $\mathrm{ATL}_{\sqsubseteq}$ and $\mathrm{ATL}_{\sqsubseteq}$ with $\langle . \sqsubseteq . \rangle$ chains

Conclusion

Coalitions combine the powers of their members:

$\langle\langle\Gamma\rangle\rangle\varphi$ – $\Gamma \subseteq Ag$ can enforce φ together.

We examine how powers can be distributed:

$\langle i \sqsubseteq \Gamma\rangle\varphi$ The powers of i can be divided among its sub-agents Γ so that the powers of $j \in \Gamma$, possibly combined with those of other agents, satisfy φ.

Example (purchase with delivery) vendor = salesperson SP + delivery team DT:

$$\langle \text{vendor} \sqsubseteq \text{SP}, \text{DT}\rangle \left(\begin{array}{l} \langle\langle \text{SP}, \text{customer}\rangle\rangle \Diamond \text{purchase agreement} \wedge \\ [\![\text{SP}, \text{others}]\!] \Box (\text{purchase agreement} \Rightarrow \langle\langle \text{DT}, \text{customer}\rangle\rangle \circ \text{delivery}) \end{array} \right)$$

is more informative than

$$\langle\langle \text{vendor}, \text{customer}\rangle\rangle \Diamond \text{purchase agreement} \wedge$$
$$[\![\text{others}]\!] \Box (\text{purchase agreement} \Rightarrow \langle\langle \text{vendor}, \text{customer}\rangle\rangle \circ \text{delivery})$$

Preliminaries: ATL on Concurrent Game Models (CGMs)

$Ag = \{1, \ldots, N\}$ - agents

$\langle W, \langle Act_i : i \in Ag \rangle, o \rangle$ - CGS for Ag

$W \neq \emptyset$ - the statespace;

$Act_i \neq \emptyset$ - the actions of agent $i \in Ag$; $\prod\limits_{i \in \Gamma} Act_i$ is written Act_Γ;

$o : W \times Act_{Ag} \to W$ - transition (outcome) function.

$AP \neq \emptyset$ - atomic propositions

$\langle W, \langle Act_i : i \in Ag \rangle, o, V \rangle$ - CGM for Ag and AP

$V \subseteq W \times AP$ - valuation.

$s_i : W^+ \to Act_i$ - a strategy for $i \in Ag$; $s_\Gamma = \langle s_i : i \in \Gamma \rangle \in S_\Gamma$

$$\mathrm{out}(w, s_\Gamma) = \left\{ \underline{w} \in W^\omega : \begin{array}{l} \underline{w}^0 = w, \underline{w}^{k+1} = o(\underline{w}^k, s_\Gamma(\underline{w}[0..k]) \cdot b^k_{Ag \setminus \Gamma}), \\ b^k_{Ag \setminus \Gamma} \in Act_{Ag \setminus \Gamma}, k < \omega \end{array} \right\}.$$

ATL on CGMs

$M = \langle W, \langle Act_i : i \in Ag \rangle, o, V \rangle$

$\varphi, \psi ::= \bot \mid p \mid (\varphi \Rightarrow \psi) \mid \langle\!\langle \Gamma \rangle\!\rangle \theta \qquad \theta ::= \circ\varphi \mid (\varphi \mathsf{U} \psi) \mid \neg(\varphi \mathsf{U} \psi)$

$$
\begin{array}{lll}
M, w \not\models \bot & & \\
M, w \models p & \text{iff} & V(w, p) \\
M, w \models \varphi \Rightarrow \psi & \text{iff} & M, w \not\models \varphi \text{ or } M, w \models \psi \\
M, w \models \langle\!\langle \Gamma \rangle\!\rangle \theta & \text{iff} & (\exists s_\Gamma \in S_\Gamma)(\forall \underline{w} \in \mathrm{out}(w, s_\Gamma)) M, \underline{w} \models \theta
\end{array}
$$

$\neg, \vee, \wedge, \Leftrightarrow, \top \ldots$, combinations of $\langle\!\langle . \rangle\!\rangle$ and $[\![.]\!]$ and $\circ$, $\Diamond$, $\Box$ - as usual.

Γ-to-i homomorphisms

$M = \langle W, \langle Act_i : i \in Ag \rangle, o, V \rangle$ for AP and Ag

$i \in Ag$, $\Gamma \neq \emptyset$, $\Gamma \cap Ag = \emptyset$, $Ag' = (Ag \setminus \{i\}) \cup \Gamma$

$M' = \langle W', \langle Act'_i : i \in Ag' \rangle, o', V' \rangle$ for AP and Ag'

$h : \prod\limits_{j \in \Gamma} Act'_j \to Act_i$ is a Γ-to-i homomorphism from M' to M, if

(i) $W' = W$, $V' = V$ and $Act_j = Act'_j$ for $j \in Ag \setminus \{i\}$;

(ii) $o'(w, a) = o(w, a_{Ag \setminus \{i\}} \cdot h(a_\Gamma))$ for all $w \in W$ and $a \in Act'_{Ag'}$;

(iii) $\operatorname{ran} h = Act_i$.

- h specifies how the members of Γ choose an action $a_i = h(a_\Gamma)$ on behalf of i.
- Successor states are determined from a_i and the actions a_j of the agents $j \in Ag \setminus \{i\}$ as in M.
- h is surjective, i.e., Γ can choose any $a_i \in Act_i$.

The Definition of $\langle i \sqsubseteq \Gamma \rangle \varphi$

$M = \langle W, \langle Act_i : i \in Ag \rangle, o, V \rangle$ for AP and Ag

$i \in Ag$, $\Gamma \neq \emptyset$, $\Gamma \cap Ag = \emptyset$, $Ag' = (Ag \setminus \{i\}) \cup \Gamma$

$M, w \models \langle i \sqsubseteq \Gamma \rangle \varphi$ if there exists an M' s.t. $M', w \models \varphi$
and a Γ-to-i homomophism from M' to M.

$\mathrm{ATL}_{\sqsubseteq}$ is the extension of ATL by $\langle . \sqsubseteq . \rangle$.

Some valid refinements written using $\langle i \sqsubseteq \Gamma \rangle \varphi$

The powers of Γ are equal to those of i:

$\models_{\mathrm{ATL}_{\sqsubseteq}} \langle\langle i \rangle\rangle \varphi \Leftrightarrow \langle i \sqsubseteq \Gamma \rangle \langle\langle \Gamma \rangle\rangle \varphi;$ In general, $\models_{\mathrm{ATL}_{\sqsubseteq}} \varphi \Leftrightarrow \langle i \sqsubseteq \Gamma \rangle [\Gamma / i] \varphi.$

Unanimity:

$$\models_{\mathrm{ATL}_{\sqsubseteq}} \langle\langle i \rangle\rangle \varphi \wedge \neg \langle\langle \emptyset \rangle\rangle \varphi \Rightarrow \langle i \sqsubseteq \Gamma \rangle \left(\langle\langle \Gamma \rangle\rangle \varphi \wedge \bigwedge_{\Delta \subsetneq \Gamma} \neg \langle\langle \Delta \rangle\rangle \varphi \right).$$

Simple Majority:

$$\models_{\mathrm{ATL}_{\sqsubseteq}} \langle\langle i \rangle\rangle \varphi \wedge \neg \langle\langle \emptyset \rangle\rangle \varphi \Rightarrow \langle i \sqsubseteq \Gamma \rangle \Big(\bigwedge_{\substack{\Delta \subseteq \Gamma \\ |\Delta| > |\Gamma \setminus \Delta|}} \langle\langle \Delta \rangle\rangle \varphi \wedge \bigwedge_{\substack{\Delta \subseteq \Gamma \\ |\Delta| \leq |\Gamma \setminus \Delta|}} \neg \langle\langle \Delta \rangle\rangle \varphi \Big).$$

More axioms about refinements in terms of $\langle i \sqsubseteq \Gamma \rangle \varphi$

Chaining refinements

$$\models_{\mathrm{ATL}_{\sqsubseteq}} \langle i \sqsubseteq j, k \rangle \langle k \sqsubseteq l, m \rangle \varphi \Rightarrow \langle i \sqsubseteq j, l, m \rangle \varphi.$$

Let

$$[i \sqsubseteq \Gamma]\varphi \rightleftharpoons \neg \langle i \sqsubseteq \Gamma \rangle \neg \varphi.$$

The better of two worlds:

$$\models_{\mathrm{ATL}_{\sqsubseteq}} \langle i \sqsubseteq j, k \rangle \varphi \wedge [i \sqsubseteq j, k]\psi \Rightarrow \langle i \sqsubseteq j, k \rangle (\varphi \wedge \psi \wedge [k/j, j/k]\psi)$$

because

$\models_{\mathrm{ATL}_{\sqsubseteq}} [i \sqsubseteq \Gamma]\varphi \Leftrightarrow [i \sqsubseteq \Gamma][\pi(j)/j : j \in \Gamma]\varphi$ for any permutation π of Γ.

More axioms and a rule about $\langle . \sqsubseteq . \rangle$

For fixed i and Γ $\langle i \sqsubseteq \Gamma \rangle$ is a $\mathbf{KD}$-modality:

$\mathbf{K}$ $\quad [i \sqsubseteq \Gamma](\varphi \Rightarrow \psi) \Rightarrow ([i \sqsubseteq \Gamma]\varphi \Rightarrow [i \sqsubseteq \Gamma]\psi)$

$\mathbf{D}$ $\quad [i \sqsubseteq \Gamma]\varphi \Rightarrow \langle i \sqsubseteq \Gamma \rangle \varphi$

$\varphi \Leftrightarrow \langle i \sqsubseteq \Gamma \rangle [\Gamma/i]\varphi$ can be viewed as a variant of $\mathbf{T}$.

Combining refinements by conjunction:

$$(\wedge_{\sqsubseteq}) \quad \langle i \sqsubseteq \Gamma_1 \rangle \varphi_1 \wedge \langle i \sqsubseteq \Gamma_2 \rangle \varphi_2 \Leftrightarrow$$
$$\langle i \sqsubseteq \Gamma_1 \times \Gamma_2 \rangle ([\{j\} \times \Gamma_2/j : j \in \Gamma_1]\varphi_1 \wedge [\Gamma_1 \times \{j\}/j : j \in \Gamma_2]\varphi_2)$$

Introducing negative occurrences of $\langle . \sqsubseteq . \rangle$ (by monotonicity and $\mathbf{T}$):

$\mathsf{c}(\varphi) = \{\Gamma : \langle\langle \Gamma \rangle\rangle$ occurs in $\varphi\}$ - the coalitions mentioned in φ.

$$(\langle . \sqsubseteq . \rangle_L) \qquad \frac{\varphi \Rightarrow \psi}{\langle i \sqsubseteq \Gamma \rangle \varphi \Rightarrow [i/\Gamma]\psi} \qquad \mathsf{c}(\psi) \subseteq \{\Delta : \Gamma \cap \Delta = \emptyset \text{ or } \Gamma \subseteq \Delta\}$$

On formalizations of delegation

• Delegation is often in the form of granting access.
E.g., delegates may receive control of designated parts of the system state.

• Alternatively, delegates may receive control of designated actions.

• The giver and the receiver of control co-exist.
Therefore refinement $\langle i \sqsubseteq \Gamma \rangle$ is not i delegating to Γ!

• Sometimes sub-agents are given assignments as opposed to powers.

• Delegates' actions may be monitored and directed by the co-ordinating agent; this is more like scheduling outsourced services and not delegating powers.

$\langle i \sqsubseteq \Gamma \rangle \varphi$ means that i is supplanted by Γ in scenario φ.

In φ, i can still be refered to as Γ, since the combined powers of Γ equal those of i.

Expressing delegation by $\langle i \sqsubseteq \Gamma \rangle \varphi$

To delegate a strategic ability φ, φ needs to be alienable.

$$\langle\langle i \rangle\rangle \varphi \wedge \langle\langle i \rangle\rangle \psi \wedge \langle i \sqsubseteq j, i' \rangle (\langle\langle j \rangle\rangle \varphi \wedge \langle\langle i' \rangle\rangle \psi \wedge \neg \langle\langle i' \rangle\rangle \neg \varphi)$$

- i's powers φ, ψ are divided between a j, who assumes φ, and an i', who assumes the remaining powers ψ of i:

Delegating to an existing agent j:

$$\langle\langle i, j \rangle\rangle \varphi \wedge \neg \langle\langle j \rangle\rangle \varphi \wedge \langle i \sqsubseteq i', j' \rangle (\langle\langle j, j' \rangle\rangle \varphi \wedge \neg \langle\langle i' \rangle\rangle \varphi)$$

- The powers of i are passed to j, by means of a virtual sub-agent j' of i's. $\{j, j'\}$ is j with its powers enhanced.

In the scope of $\langle i \sqsubseteq i', j' \rangle$, the whole of i can be refered to as $\{i', j'\}$.

Just writing $\langle\langle i \rangle\rangle \langle\langle j \rangle\rangle \varphi$ says that j receives the power to enforce φ thanks to i's action. This power need not belong to i.

Model-checking $\langle . \sqsubseteq . \rangle$-flat $\mathrm{ATL}_{\sqsubseteq}$

$\langle . \sqsubseteq . \rangle$-flat $\mathrm{ATL}_{\sqsubseteq}$ is the subset of $\mathrm{ATL}_{\sqsubseteq}$, with no occurrences of $\langle . \sqsubseteq . \rangle$ in the scope of $\langle . \sqsubseteq . \rangle$.

Model-checking boils down to deciding

$$M, w \models \langle i \sqsubseteq \Gamma \rangle \varphi,$$

for $\langle . \sqsubseteq . \rangle$-free φ.

This reduces to satisfiability in the $\langle\langle . \rangle\rangle \circ$-subset of ATL without $\langle . \sqsubseteq . \rangle$ (equivalently, Pauly's Coalition Logic), which is known to be decidable.

$$M, w \models \langle i \sqsubseteq \Gamma \rangle \varphi$$

$M = \langle W, \langle Act_i : i \in Ag \rangle, o, V \rangle$

φ - b.c. of formulas of the form $\langle\langle . \rangle\rangle \circ \chi$, χ - b.c. of atomic propositions.

$M' = \langle W, \langle Act_i : i \in Ag' \rangle, o', V \rangle$

$h : M' \to M$ - a Γ-to-i homomorphism

$\langle\langle \Delta \rangle\rangle \circ \chi \in \mathrm{Subf}(\varphi)$, $\qquad \Delta \subseteq Ag' = Ag \setminus \{i\} \cup \Gamma$

$M', w \models \langle\langle \Delta \rangle\rangle \circ \chi$ if there exist $a_{\Delta \setminus \Gamma}$ and $a_{\Delta \cap \Gamma}$ such that for all $b_{\Gamma \setminus \Delta}$ coalition $\Delta \setminus \Gamma \cup \{i\}$ can achieve χ in M by playing $a_{\Delta \setminus \Gamma} \cdot h(a_{\Delta \cap \Gamma} \cdot b_{\Gamma \setminus \Delta})$.

Given a fixed $a_{\Delta \setminus \Gamma}$, this means

$$h(a_{\Delta \cap \Gamma} \cdot b_{\Gamma \setminus \Delta}) \in \underbrace{\{a_i \in Act_i : \forall c_{Ag \setminus (\Delta \cup \{i\})} M, o(w, a_i \cdot a_{\Delta \setminus \Gamma} \cdot c_{Ag \setminus (\Delta \cup \{i\})}) \models \chi\}}_{A_{i, a_{\Delta \setminus \Gamma}, w, \chi} \subseteq Act_i}$$

$M', w \models \langle\!\langle \Delta \rangle\!\rangle \circ \chi$ is equivalent to $\exists a_{\Delta \setminus \Gamma} \underbrace{\exists a_{\Delta \cap \Gamma} \forall b_{\Gamma \setminus \Delta} (h(a_{\Delta \cap \Gamma} \cdot b_{\Gamma \setminus \Delta}) \in A_{i, a_{\Delta \setminus \Gamma}, w, \chi})}_{\overline{M} \models \langle\!\langle \Delta \cap \Gamma \rangle\!\rangle \circ A_{i, a_{\Delta \setminus \Gamma}, w, \chi}}$.

$\overline{M} = \langle \overline{W}, \langle \overline{Act}_i : i \in \Gamma \rangle, \overline{o}, \overline{V} \rangle$ for $\overline{Ag} = \Gamma$ and $\overline{AP} = Act_i$.

$\overline{W} = Act_i \cup \{ \underbrace{w_0}_{\notin Act_i} \}$

$\overline{V}(w, a) \leftrightarrow w = a, \quad a \in Act_i$

$\overline{Act}_j = Act'_j$ (from M'), $\quad j \in \Gamma$

$\overline{o}(w_0, a) = h(a)$ for all $a \in \overline{Act}_\Gamma$

Then $M, w \models \langle i \sqsubseteq \Gamma \rangle \langle\!\langle \Delta \rangle\!\rangle \circ \chi$ is equivalent to the existence of an $\overline{M}$ of the above form and a $w_0 \in \overline{W}$ such that

$$\overline{M}, w_0 \models \bigvee_{a_{\Delta \setminus \Gamma} \in Act_{\Delta \setminus \Gamma}} \langle\!\langle \Delta \cap \Gamma \rangle\!\rangle \circ \bigvee_{a_i \in A_{i, a_{\Delta \setminus \Gamma}, w, \chi}} a_i.$$

i.e., to the satisfiability of the above formula.

$\overline{M} = \langle \overline{W}, \langle \overline{Act}_i : i \in \Gamma \rangle, \overline{o}, \overline{V} \rangle$ for agents Γ and $\overline{AP} = Act_i$

$\overline{W} = Act_i \cup \{ \underbrace{w_0}_{\notin Act_i} \}$

$\overline{V}(w, a) \leftrightarrow w = a$ for $a \in \overline{AP} = Act_i$

$\overline{Act}_j = Act'_j$ (from M') for $j \in \Gamma$

$\overline{o}(w_0, a) = h(a)$ for all $a \in \overline{Act}_\Gamma$; $\overline{o}(w, .)$ is irrelevant for $w \neq w_0$.

Then, at w_0, $\overline{M}$ satisfies

$$(*) \qquad \langle\langle \emptyset \rangle\rangle \bigvee_{a \in Act_i} a \wedge \bigwedge_{a,b \in Act_i, a \neq b} \langle\langle \emptyset \rangle\rangle \circ \neg (a \wedge b) \wedge \bigwedge_{a \in Act_i} \langle\langle \Gamma \rangle\rangle \circ a,$$

Consider a translation t which replaces any $\langle\langle \Delta \rangle\rangle \circ \chi \in \mathrm{Subf}(\varphi)$ by its corresponding

$$\bigvee_{a_{\Delta \setminus \Gamma} \in Act_{\Delta \setminus \Gamma}} \langle\langle \Delta \cap \Gamma \rangle\rangle \circ \bigvee_{a_i \in A_{i, a_{\Delta \setminus \Gamma}, w, \chi}} a_i.$$

Then $M, w \models \varphi$ is equivalent to $(\exists \overline{M})\, \overline{M}, w_0 \models (*) \wedge \mathrm{t}(\varphi)$.

Model-checking $\langle . \sqsubseteq . \rangle$-flat $\mathrm{ATL}_{\sqsubseteq}$

The algorithm extends to $\langle . \sqsubseteq . \rangle$ with arbitrary ATL formulas as the argument, but in a rather "brute-force" fashion.

It is good only to conclude decidability of model-checking in principle.

The algorithm extends to formulas of the form

$$\langle i_1 \sqsubseteq \Gamma_1 \rangle \ldots \langle i_K \sqsubseteq \Gamma_K \rangle \varphi$$

too. Such formulas can express refining several parties simultaneously.

Concluding Remarks

All we know so far is the decidability of model-checking (of a subset!)

• More efficient model-checking?

• Altermating nestings of $\langle . \sqsubseteq . \rangle$s?

• A proof system? Our axioms and rule are sufficient for validity with negative occurrences of $\langle . \sqsubseteq . \rangle$ only.

• $\langle . \sqsubseteq . \rangle$ can be defined without reference to the existence of homomorphisms. In predicate logic, the definition is in the $\exists^*\forall^*\exists^*$ class, over many-sorted models with Act_i, $i \in Ag$, as the sorts.

• The finite model property? The FMP for the$\langle\!\langle . \rangle\!\rangle\circ$-subset of $\mathrm{ATL}_{\sqsubseteq}$?

• How to define Γ-to-i HMMs for incomplete information? E.g., should sub-agents be allowed to have 'additional' local state?

The End