Reasoning about Knowledge and Strategies: Epistemic Strategy Logic

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Strategic Reasoning – 5 April 2014
Overview

1 Motivation and Background
   ▶ logics for reasoning about strategies and knowledge

2 Epistemic Strategy Logic
   ▶ semantics
   ▶ syntax

3 Main Contribution
   ▶ model checking ESL is no harder than SL

4 Imperfect Information
   ▶ benefits of combining epistemic and strategy modalities

5 Conclusions and Future Work
Motivation and Background

Logics of strategic abilities

- Logics for strategic reasoning are a thriving area of research in AI and MAS.
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- Two lines of research:
  1. Multi-modal logics to formalise strategic abilities and behaviours of individual agents and groups:
     - Alternating-time Temporal Logic [AHK02]
     - Coalition Logic [Pau02]
     - Strategy Logic [CHP10, MMV10]
  2. Extensions of logics for reactive systems with epistemic operators to reason about the knowledge agents have of the system’s evolution:
     - Combinations of CTL and LTL with multi-modal epistemic logic $S5_n$ [HV86, HV89, FHMV95]
     - Successfully applied to MAS specification and verification [GvdM04, KNN+08, LQR09]
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- Along these lines, [vdHW03] introduced ATEL.
  - spawned a wealth of contributions:
    - imperfect information/uniform strategies [Sch04, JvdH04]
    - constructive knowledge [JÅ07]
    - irrevocable/feasible strategies [AGJ07, Jon03]
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Epistemic Strategy Logic = strategies + knowledge

- Topic of interest [HvdM14b, HvdM14a]
The Prisoner’s Dilemma

Games in Normal Form

- Anne and Bob can either Cooperate or Defect
- payoff ordering: \( a > b > c > d \)

\[
\begin{array}{c|cc}
\text{Bob} & \text{Cooperate} & \text{Defect} \\
\hline
\text{Cooperate} & b, b & d, a \\
\text{Defect} & a, d & c, c
\end{array}
\]

- can Anne achieve payoff \( a \)?
- does Anne know whether she can achieve payoff \( a \)?
- does Bob know whether Anne has a strategy to achieve payoff \( a \)?
- do Anne and Bob know \((de \ dicto)\) whether they can reach a Nash equilibrium?
- are there strategies such that Anne and Bob know \((de \ re)\) that they can reach a Nash equilibrium?
Epistemic Concurrent Game Models

Agents

We adopt an agent-oriented perspective.

**Definition (Agent)**

An *agent* $i$ is

- situated in some *local state* $l_i \in L_i$ and . . .
- performs the *actions* in $\text{Act}_i$
- . . . according to her *protocol function* $\text{Pr}_i : L_i \mapsto 2^{\text{Act}_i}$

The setting is reminiscent of the *interpreted systems semantics* for MAS [FHMV95].

**Example (Prisoner's Dilemma)**

Agent $\text{Anne} = \langle L_A, \text{Act}_A, \text{Pr}_A \rangle$ is defined as

- $L_A = \{\epsilon_A, a, b, c, d\}$
- $\text{Act}_A = \{C, D, *\}$, where * is the *skip* action
- $\text{Pr}_A(\epsilon_A) = \{C, D\}$ and $\text{Pr}_A(a) = \text{Pr}_A(b) = \text{Pr}_A(c) = \text{Pr}_A(d) = \{*\}$

The definition of agent *Bob* is symmetric.
Epistemic Concurrent Game Models

ECGM

The interactions amongst agents generate ECGM.

- related to CGS [AHK02, MMV10] and AETS [vdHW03]
- global states are not primitive: \( s = \langle l_0, \ldots, l_\ell \rangle \in G = \Pi_{i \in Ag} L_i \)
- joint actions are tuples \( \sigma = \langle \sigma_0, \ldots, \sigma_\ell \rangle \in Act = \Pi_{i \in Ag} Act_i \)

**Definition (ECGM)**

Given
- a set \( Ag = \{i_0, \ldots, i_\ell\} \) of agents
- a set \( AP \) of atomic propositions

an ECGM \( \mathcal{P} \) includes
- a finite set \( I \subseteq G \) of initial global states
- a transition function \( \tau : G \times Act \rightarrow G \)
- an interpretation \( \pi : AP \rightarrow 2^G \) of atomic propositions

- the epistemic indistinguishability relation is not primitive: \( s \sim_i s' \) iff \( l_i = l_i' \)
The Prisoner's Dilemma as an ECGM

Let $AP = \{a_i, b_i, c_i, d_i\}$ for $i \in \{A, B\}$.

Example (ECGM $\mathcal{P}_{pd}$)

For the set $Ag = \{A, B\}$ of agents, the prisoner’s dilemma ECGM $\mathcal{P}_{pd}$ includes

- the set $I = \{s_0\}$ of initial states, with $s_0 = (\epsilon_A, \epsilon_B)$
- the transition function $\tau$, given as
  - $\tau(s_0, (C, C)) = (b, b)$
  - $\tau(s_0, (C, D)) = (d, a)$
  - $\tau(s_0, (D, C)) = (a, d)$
  - $\tau(s_0, (D, D)) = (c, c)$
  - $\tau(s, (\ast, \ast)) = s$, for every state $s$ different from $s_0$
- the interpretation $\pi$ s.t. a state $(l_A, l_B)$ belongs to $\pi(p_i)$ iff $l_i = p$. 
Epistemic Strategy Logic

ESL

ESL extends SL with epistemic operators $K_i$ for individual knowledge.

- we introduce a set $Var_i$ of strategy variables for each agent $i \in Ag$

**Definition (ESL)**

ESL formulas are defined in BNF as follows:

$$\phi ::= p \mid \neg \phi \mid \phi \rightarrow \phi \mid X\phi \mid \phi U \phi \mid \exists x_i \phi \mid K_i \phi$$

- we consider a multi-agent setting ($\neq [CHP10]$)
- the language does not include the binding operator $(\alpha, x)$ ($\neq [MMV10]$)

The questions above can be recast as model checking problems:

$$\mathcal{P}_{pd} \models ? \exists x_A F a_A$$

$$\mathcal{P}_{pd} \models K_A(\exists x_A F a_A \lor \neg \exists x_A F a_A)$$

$$\mathcal{P}_{pd} \models K_B(\exists x_A F a_A \lor \neg \exists x_A F a_A)$$
Epistemic Concurrent Game Models

Strategies

**Definition (Strategy)**

An $A$-strategy is a mapping $f_A : G^+ \mapsto Act_A$ from finite sequences of states to enabled $A$-actions.

- a run $\lambda$ is a sequence $s^0 \rightarrow s^1 \rightarrow \ldots$ of global states
- a run $\lambda$ belongs to outcome $out(s, f_A)$ iff $\lambda(i + 1) \in \hat{\tau}(\lambda(i), f_A(\lambda[\ldots, i]))$
  \[ \Rightarrow \text{ a group strategy is really the composition of its members' strategies} \]
- an assignment $\chi$ maps each agent $i \in Ag$ to an $i$-strategy $f_i$
  \[ f^\chi \text{ is the } Ag\text{-strategy } \chi(i_0) \times \ldots \times \chi(i_\ell) \]

**Definition (Satisfaction)**

An ECGM $\mathcal{P}$ satisfies an ESL formula $\varphi$ in a state $s$ for an assignment $\chi$, iff

\[
\begin{align*}
(\mathcal{P}, s, \chi) \models p & \quad \text{iff} \quad s \in \pi(p) \\
(\mathcal{P}, s, \chi) \models X\psi & \quad \text{iff} \quad \text{for } \lambda = out(s, f^\chi), (\mathcal{P}, \lambda(1), \chi) \models \psi \\
(\mathcal{P}, s, \chi) \models \psi U\psi' & \quad \text{iff} \quad \text{for } \lambda = out(s, f^\chi) \text{ there is } k \geq 0 \text{ s.t. } (\mathcal{P}, \lambda(k), \chi) \models \psi' \\
& \quad \text{and } 0 \leq j < k \text{ implies } (\mathcal{P}, \lambda(j), \chi) \models \psi \\
(\mathcal{P}, s, \chi) \models \exists x_i \psi & \quad \text{iff} \quad \text{there exists an } i\text{-strategy } f_i \text{ s.t. } (\mathcal{P}, s, \chi_f^i) \models \psi \\
(\mathcal{P}, s, \chi) \models K_i \psi & \quad \text{iff} \quad \text{for every } s \in S, s \sim_i s' \text{ implies } (\mathcal{P}, s', \chi) \models \psi
\end{align*}
\]
Expressiveness
Knowledge of Nash Equilibria

• given an \( n \)-player game in normal form with payoff ordering \( a_1 > \ldots > a_k \), define

\[
\psi_{NE} \quad ::= \quad \bigwedge_{i=1}^n \bigwedge_{j=1}^k \left( \bigwedge_{i=1}^{i-1} \neg \exists y_j Xa_j \right) \land \exists y_i Xa_i \rightarrow Xa_i
\]

Proposition

\((P_{pd}, s_0, \chi) \models \psi_{NE} \iff (\chi(1)(s_0), \ldots, \chi(n)(s_0)) \text{ is a Nash equilibrium}\)

• for the prisoner’s dilemma,

\((P_{pd}, s_0, \chi) \models \psi_{NE} \iff (\chi(1)(s_0), \chi(2)(s_0)) \text{ is a Nash equilibrium} \iff \chi(1)(s_0) = \chi(2)(s_0) = D\)

The questions above can be recast as model checking problems:

\[
P_{pd} \models \text{?} \quad K_A \exists x_A, x_B \psi_{NE} \land K_B \exists x_A, x_B \psi_{NE}
\]

\[
P_{pd} \models \text{?} \quad \exists x_A, x_B (K_A \psi_{NE} \land K_B \psi_{NE})
\]
Expressiveness

Knowledge de re v. Knowledge de dicto

- knowledge de re $\Rightarrow$ knowledge de dicto:

$$\models \exists x_i K_j \phi \rightarrow K_j \exists x_i \phi$$

also, knowledge de dicto $\Rightarrow$ knowledge de re:

$$\models K_j \exists x_i \phi \rightarrow \exists x_i K_j \phi$$

indeed, agents have perfect information of the game

- individual strategies depend on global states [JvdH04]
Model Checking ESL

Theorem (Hardness)

The model checking problem for ESL is Non-ElementarySpace-hard.

- reduction to satisfiability for quantified propositional temporal logic (QPTL)
- differently from [MMV10] the syntax does not include the binding operator

Theorem (Completeness)

The model checking problem for ESL is PTime-complete w.r.t. the size of the model and Non-Elementary w.r.t. the size of the formula.

- reduction to non-emptyness for alternating tree automata [MMV10]

⇒ The model checking problem is no harder for ESL than for SL.
Imperfect Information

- Bob chooses secretly between 0 and 1
- at the next step Anne also chooses between 0 and 1
- Anne wins the game iff the values provided by the two players coincide
- the dotted line indicates epistemic indistinguishability

- Anne knows that there exists a strategy to win the game . . .
  . . . however, she is not able to point this strategy out

⇒ Anne has imperfect information of the game
Under imperfect information, strategies depend on the local state of agents only.

**Definition (Uniform Strategies [JvdH04])**

A (positional) $i$-strategy is *uniform* iff for all states $s, s'$, $s \sim_i s'$ implies $f_i(s) = f_i(s')$.

- Anne knows that there exists a strategy to win the game . . .
  
  $$(Q, s_{\lambda 0}) \models_{ii} K_A \exists x_A \ X \text{ win}$$

  . . . however, there is no strategy that she knows to be winning:

  $$(Q, s_{\lambda 0}) \nmodels_{ii} \exists x_A K_A \ X \text{ win}$$
Epistemic modalities allow us to recover some fixed-point characterisations of ATL operators.

- for $A = \{i_0, \ldots, i_\ell\}$, $\vec{x}_A = x_{i_0}, \ldots, x_{i_\ell}$ and $\bar{A} = Ag \setminus A$, define

\[ \langle\langle A \rangle\rangle \phi := \exists \vec{x}_A \forall \vec{x}_{\bar{A}} \phi \]

- under imperfect information we have that

\[ \langle\langle A \rangle\rangle G \phi \not\iff \phi \land \langle\langle A \rangle\rangle X \langle\langle A \rangle\rangle G \phi \]
\[ \langle\langle A \rangle\rangle F \phi \not\iff \phi \lor \langle\langle A \rangle\rangle X \langle\langle A \rangle\rangle F \phi \]
\[ \langle\langle A \rangle\rangle \phi U \phi' \not\iff \phi' \lor (\phi \land \langle\langle A \rangle\rangle X \langle\langle A \rangle\rangle \phi U \phi' \land K_i(\langle\langle A \rangle\rangle \phi U \phi' \rightarrow \phi U \phi')) \]

- by using epistemic modalities we can recover fixed-point characterisations:

\[ \langle\langle i \rangle\rangle G \phi \iff \phi \land \langle\langle i \rangle\rangle X \langle\langle i \rangle\rangle (G \phi \land K_i(\langle\langle i \rangle\rangle G \phi \rightarrow G \phi)) \]
\[ \langle\langle i \rangle\rangle F \phi \iff \phi \lor \langle\langle i \rangle\rangle X \langle\langle i \rangle\rangle (F \phi \land K_i(\langle\langle i \rangle\rangle F \phi \rightarrow F \phi)) \]
\[ \langle\langle i \rangle\rangle \phi U \phi' \iff \phi' \lor (\phi \land \langle\langle i \rangle\rangle X \langle\langle i \rangle\rangle (\phi U \phi' \land K_i(\langle\langle i \rangle\rangle \phi U \phi' \rightarrow \phi U \phi')))) \]
Conclusions

Results:
• ESL: a logic for reasoning about knowledge and strategies in a multi-agent setting
• the model checking problem is no harder than for SL
• under imperfect information ESL allows us to recover the fixed-point characterisation of ATL operators

and Future Work:
• fragments of ESL: better computational complexity?
• epistemic operators for group knowledge (distributed, common knowledge, etc.)
• imperfect information
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