Automata Techniques for Epistemic Protocol Synthesis

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Two important approaches to add dynamics to Epistemic Logics:

**Epistemic Temporal Logics**
A model usually consists of:

- **Dynamics**: A finite transition system
- **Epistemics**: Observational equivalences on states.

**Dynamic Epistemic Logics**
Much finer way to describe the events and how they are perceived.

- **Epistemics**: Epistemic models and event models to represent
  - possible worlds, and how they are perceived,
  - possible events, and how they are perceived.
- **Dynamics**: Update product between epistemic and event models
What about strategizing/planning?

In the context of ETL:
- Has been, and still is, much studied
- Many decidability/complexity results
- Rely on the fact that the set of histories is regular
  - Powerset constructions
  - Tree automata techniques

In the context of DEL:
- Very little results
- Because the set of histories is not regular in general?

In this work:
- Identify a condition for DEL-generated structures to be regular
- Use automata techniques to tackle planning problems in DEL
1 Dynamic Epistemic Logic (DEL)

2 From DEL to regular structures

3 Epistemic planning and epistemic protocol synthesis
An example: Alice and Bob toss a coin in the dark.

The initial epistemic state:
- The coin is on heads
- Alice and Bob both ignore it
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The initial epistemic state:
- The coin is on heads
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The event:
- The light turns on briefly
- Alice sees that it is heads
- Short-sighted Bob sees tails
An example: Alice and Bob toss a coin in the dark.

The initial epistemic state

The event
An example: Alice and Bob toss a coin in the dark.

The initial epistemic state

\[ (w, e), (w', e') \]

The event

\[ (w, e) \rightarrow (w', e') \]

The resulting epistemic state
## Epistemic models

### Epistemic language $\mathcal{L}^{EL}$

$$\varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid K_i \varphi \quad (p \in AP, i \in Ag)$$

### Epistemic models

$$\mathcal{M} = (W, \{R_i\}_{i \in Ag}, V)$$

- $W$ is a non-empty finite set of possible worlds,
- $R_i \subseteq W \times W$ is an accessibility relation for agent $i$,
- $V : AP \rightarrow 2^W$ is a valuation function.

### Semantics of $\mathcal{L}^{EL}$

- $\mathcal{M}, w \models p$ if $w \in V(p)$
- $\mathcal{M}, w \models \neg \varphi$ if $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \lor \psi$ if $\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i \varphi$ if $\mathcal{M}, w' \models \varphi$ for all $w' \in R_i(w)$
Event models

\[ \mathcal{E} = (E, \{R_i\}_{i \in Ag}, \text{pre}, \text{post}) \]

- \( E \) is a non-empty finite set of possible events,
- \( R_i \subseteq E \times E \) is an accessibility relation on \( E \) for agent \( i \),
- \( \text{pre} : E \rightarrow \mathcal{L}^{EL} \) is a precondition function and
- \( \text{post} : E \rightarrow AP \rightarrow \mathcal{L}^{EL} \) is a postcondition function.

Propositional event models

Pre and post-conditions are propositional.
Update product and DEL-generated structures

Product of $\mathcal{M} = (W, \{R_i\}_{i \in Ag}, V)$ and $\mathcal{E} = (E, \{R_i\}_{i \in Ag}, \text{pre}, \text{post})$

$$\mathcal{M} \otimes \mathcal{E} = (W^\otimes, \{R_i^\otimes\}_{i \in Ag}, V^\otimes)$$

$$W^\otimes = \{(w, e) \in W \times E \mid \mathcal{M}, w \models \text{pre}(e)\},$$

$$R_i^\otimes(w, e) = \{(w', e') \in W^\otimes \mid w' \in R_i(w) \text{ and } e' \in R_i(e)\},$$

$$V^\otimes(p) = \{(w, e) \in W^\otimes \mid \mathcal{M}, w \models \text{post}(e)(p)\}$$

Structure generated from $\mathcal{M}$ and $\mathcal{E}$

$$\mathcal{M}\mathcal{E}^n = \mathcal{M} \otimes \mathcal{E} \otimes \ldots \otimes \mathcal{E} \quad \text{for } n \text{ times}$$

$$\mathcal{M}\mathcal{E}^* = \bigcup_{n \geq 0} \mathcal{M}\mathcal{E}^n = (H, \{\sim_i\}_{i \in Ag}, V)$$

An element $(w, e_1, \ldots, e_n)$ of $\mathcal{M}\mathcal{E}^*$ is seen as a history $we_1 \ldots e_n$, and $we_1 \ldots e_n \sim_i w'e_1' \ldots e_n'$ if $w R_i w'$ and $e_k R_i e_k'$ for all $k$. 
Dynamic Epistemic Logic (DEL)

From DEL to regular structures

Epistemic planning and epistemic protocol synthesis

Plan

1. Dynamic Epistemic Logic (DEL)

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Definition

**Regular structures**

A relational structure \( S = (D, \{\sim_i\}_{i \in Ag}, V) \) is regular if:

- \( D \subseteq \Sigma^* \) is a **regular language** over some alphabet \( \Sigma \),
- for each \( i \in Ag \), \( \sim_i \) is a **regular relation**, and
- for each \( p \in AP \), \( V(p) \subseteq D \subseteq \Sigma^* \) is a **regular language**.

In other words, the structure is representable by finite automata.
Regular relations

A binary relation over words is **regular** if it is recognized by a synchronous transducer.

**Example: synchronous perfect recall**

- Let $\sim \subseteq \Sigma \times \Sigma$ be an accessibility relation.
- Extend it to words:
  
  $$a_1 \ldots a_n \sim a'_1 \ldots a'_n \text{ if } a_k \sim a'_k \text{ for each } k.$$  

- Recognized by:
  
  $$q_0$$
  
  if $a \sim a'$
From DEL to automata

**Theorem: from DEL to automata**

For every epistemic model $\mathcal{M}$ and propositional event model $\mathcal{E}$, $\mathcal{M}\mathcal{E}^*$ is an **automatic structure**, and we can build recognizers.

Define $\mathcal{A} = (\Sigma, Q, \delta, q_\emptyset, F)$, where

- $\Sigma = W \cup E$,
- $F = \{q_\nu \mid \nu \subseteq AP\}$, $Q = F \cup \{q_\emptyset\}$, and $\forall w \in W$, $\forall e \in E$,

\[
\delta(q_\emptyset, w) = q_\nu(w) \quad \delta(q_\emptyset, e) \text{ is undefined,} \quad \delta(q_\nu, w) \text{ is undefined}
\]

\[
\delta(q_\nu, e) = \begin{cases} q_{\nu'}, \text{ with } \nu' = \{p \mid \nu \models \text{post}(e)(p)\} & \text{if } \nu \models \text{pre}(e) \\ \text{undefined} & \text{otherwise.} \end{cases}
\]

**Fact**

$\mathcal{A}$ accepts exactly the histories in $\mathcal{M}\mathcal{E}^*$. 
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Epistemic planning

The epistemic planning problem (EPP)

Input:
- a pointed initial epistemic model \((M, w)\)
- an event model \(E\)
- a goal formula \(\varphi \in \mathcal{L}^{EL}\)

Output:
- Is there \(e_1 \ldots e_n\) s.t. \((M, w) \otimes (E, e_1) \otimes \ldots \otimes (E, e_n) \models \varphi\) ?

Example

Is there a finite sequence of events such that in the end, Alice and Bob both know how is the coin?
Epistemic planning

The epistemic planning problem (EPP)

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- a pointed initial epistemic model \((\mathcal{M}, w)\)
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- Is there \(e_1 \ldots e_n\) s.t. \(\mathcal{M}\mathcal{E}^*, we_1 \ldots e_n \models \varphi\)?

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Epistemic planning

The propositional epistemic planning problem (propositional EPP)

Input:
- a pointed initial epistemic model \((\mathcal{M}, w)\)
- a propositional event model \(\mathcal{E}\)
- a goal formula \(\varphi \in \mathcal{L}^{EL}\)

Output:
- Is there \(e_1 \ldots e_n\) s.t. \(\mathcal{M}\mathcal{E}^*, we_1 \ldots e_n \models \varphi\) ?

Example

Is there a finite sequence of events such that in the end, Alice and Bob both know how is the coin?

Theorem [Yu et al. 2013]

The propositional epistemic planning problem is decidable.
Our contribution

Theorem [Yu et al. 2013]

The propositional epistemic planning problem is decidable.

[Yu et al. 2013] prove that a finite search tree is sufficient.

We propose:

Alternative proof, based on automata techniques.
- Provides better upper bounds on the complexity.
- Builds an automaton that generates all the solution plans.
- Our approach allows to solve a much more general problem.
Uniform strategies

Use techniques developed for computing uniform strategies with rational relations.

**Uniform strategy problem (roughly):**

**Input:**
- A finite game arena
- \( n \) transducers that recognize relations between plays
- A CTL*\( K_n \) formula \( \varphi \)

**Output:**
- Is there a strategy for Player 1 that verifies \( \varphi \)?

[M., Pinchinat 2013]

The uniform strategy problem is decidable (nonelementary).
Back to epistemic planning

Take an instance \(((\mathcal{M}, w), \mathcal{E}, \varphi)\) of the propositional EPP.

- Build an automatic representation \((\mathcal{A}, \{T_i\}_{i \in \text{Ag}})\) of \(\mathcal{M}\mathcal{E}^*\)
- See \(\mathcal{A}\) as a one player game arena
- Look for a strategy (a play) that verifies \(\text{EF}\varphi\) (or \(\text{AF}\varphi\))

Because the uniform strategy problem is decidable for regular relations, we get the decidability of Propositional EPP, and more:

**Theorem**

**Arbitrary relations:** Prop. EPP is in \((d(\varphi) + 1)\)-\text{EXPTIME}

**Equivalence relations:** Prop. EPP is in \((ad(\varphi) + 1)\)-\text{EXPTIME}

**Synthesis:** Build automaton that generates all the solution plans.
Epistemic protocol synthesis in DEL

Epistemic planning

- finite sequence of events
- reach epistemic objective

Theorem

The propositional epistemic protocol synthesis problem is decidable.

Same techniques and same complexity as for epistemic planning.
Epistemic protocol synthesis in DEL

**Epistemic planning**
- finite sequence of events
- reach epistemic objective

**Epistemic protocol synthesis**
- infinite tree of events
- $\text{CTL}^* K_n$ specification

Theorem

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- finite sequence of events
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Epistemic protocol synthesis
- infinite tree of events
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Theorem
The propositional epistemic protocol synthesis problem is decidable.

Same techniques and same complexity as for epistemic planning.
Conclusion

- Connection between DEL-generated structures and regular structures
- This bridge allows us to apply existing automata techniques
- Alternative decidability proof for Propositional EPP
- Side results:
  - Improved complexity upper-bounds
  - Synthesize an automaton that recognizes the solution plans
- Same techniques apply to solve the generalized problem of Epistemic protocol synthesis
Future work

- Are these decision procedures optimal?
- Consider asynchronous variants of DEL
- Put strategic aspects in DEL.
  - By whom is an event triggered?
  - What are the agents’ objectives?

Thank you!
Semantics of $\bullet$ and $\blacklozenge$

Blue arrows: $\sim$

$\blacklozenge$: Strict quantifier

$\blacklozenge$: Full quantifier
Semantics of $\sim$ and $\sim\sim$

Blue arrows: $\sim$

$\sim$ : Strict quantifier

$\sim\sim$ : Full quantifier
Semantics of $\sqsubseteq$ and $\sqsubseteq$

Blue arrows: $\bowtie$

$\sqsubseteq$: Strict quantifier

$\sqsubseteq$: Full quantifier
Semantics of $\sqsubseteq$ and $\sqsubseteq$

Blue arrows: $\sim$

$\sqsubseteq$: Strict quantifier

$\sqsubseteq$: Full quantifier
Semantics of \( \sqsupseteq \) and \( \sqsubseteq \)

Blue arrows: \( \sim \)

\( \sqsupseteq \) : Strict quantifier

\( \sqsubseteq \) : Full quantifier