Automata Techniques for Epistemic Protocol Synthesis

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Context

Two important approaches to add dynamics to Epistemic Logics:

Epistemic Temporal Logics

A model usually consists of:

Dynamics: A finite transition system

Epistemics: Observational equivalences on states.

Dynamic Epistemic Logics

Much finer way to describe the events and how they are perceived.

Epistemics: Epistemic models and event models to represent

- possible worlds, and how they are perceived,
- possible events, and how they are perceived.

Dynamics: Update product between epistemic and event models

What about strategizing/planning?

In the context of ETL:

- Has been, and still is, much studied
- Many decidability/complexity results
- Rely on the fact that the set of histories is regular
  - Powerset constructions
  - Tree automata techniques

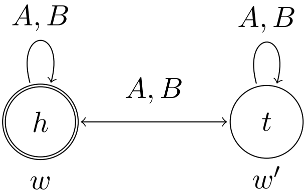
In the context of DEL:

- Very little results
- Because the set of histories is not regular in general?

In this work:

- Identify a condition for DEL-generated structures to be regular
- Use automata techniques to tackle planning problems in DEL

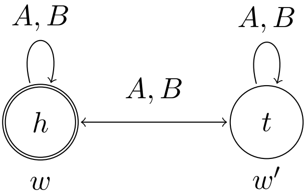
An example: Alice and Bob toss a coin in the dark.

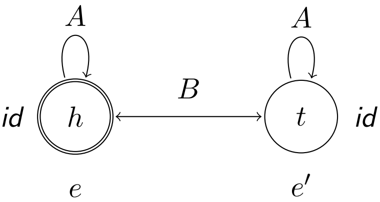


The initial epistemic state:

- The coin is on heads
- Alice and Bob both ignore it

An example: Alice and Bob toss a coin in the dark.





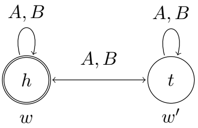
The initial epistemic state:

- The coin is on heads
- Alice and Bob both ignore it

The event :

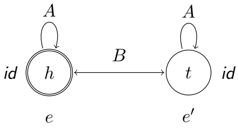
- The light turns on briefly
- Alice sees that it is heads
- Short-sighted Bob sees tails

An example: Alice and Bob toss a coin in the dark.



The initial epistemic state

$\otimes$



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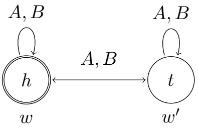

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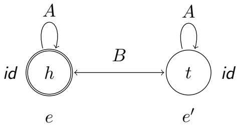
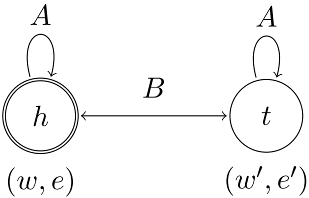


The event

$=$



The resulting epistemic state

Epistemic models

Epistemic language $\mathcal{L}^{EL}$

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid K_i\varphi \qquad (p \in AP, i \in Ag)$$

Epistemic models

$\mathcal{M} = (W, \{R_i\}_{i \in Ag}, V)$

- W is a non-empty finite set of possible worlds,
- $R_i \subseteq W \times W$ is an accessibility relation for agent i,
- $V : AP \to 2^W$ is a valuation function.

Semantics of $\mathcal{L}^{EL}$

- $\mathcal{M}, w \models p$ if $w \in V(p)$
- $\mathcal{M}, w \models \neg\varphi$ if $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \vee \psi$ if $\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ if $\mathcal{M}, w' \models \varphi$ for all $w' \in R_i(w)$

Event models

Event models

$\mathcal{E} = (\mathsf{E}, \{\mathsf{R}_i\}_{i \in Ag}, \mathsf{pre}, \mathsf{post})$

- E is a non-empty finite set of possible events,
- $\mathsf{R}_i \subseteq \mathsf{E} \times \mathsf{E}$ is an accessibility relation on E for agent i,
- $\mathsf{pre} : \mathsf{E} \to \mathcal{L}^{EL}$ is a precondition function and
- $\mathsf{post} : \mathsf{E} \to AP \to \mathcal{L}^{EL}$ is a postcondition function.

Propositional event models

Pre and post-conditions are propositional.

Update product and DEL-generated structures

Product of $\mathcal{M} = (W, \{R_i\}_{i\in Ag}, V)$ and $\mathcal{E} = (\mathsf{E}, \{\mathsf{R}_i\}_{i\in Ag}, \mathsf{pre}, \mathsf{post})$

$\mathcal{M} \otimes \mathcal{E} = (W^{\otimes}, \{R_i^{\otimes}\}_{i\in Ag}, V^{\otimes})$

$$W^{\otimes} = \{(w,e) \in W \times \mathsf{E} \mid \mathcal{M}, w \models \mathsf{pre}(e)\},$$
$$R_i^{\otimes}(w,e) = \{(w',e') \in W^{\otimes} \mid w' \in R_i(w) \text{ and } e' \in \mathsf{R}_i(e)\},$$
$$V^{\otimes}(p) = \{(w,e) \in W^{\otimes} \mid \mathcal{M}, w \models \mathsf{post}(e)(p)\}$$

Structure generated from $\mathcal{M}$ and $\mathcal{E}$

- $\mathcal{M}\mathcal{E}^n = \mathcal{M} \otimes \overbrace{\mathcal{E} \otimes \ldots \otimes \mathcal{E}}^{n \text{ times}}$
- $\mathcal{M}\mathcal{E}^* = \bigcup_{n\geq 0} \mathcal{M}\mathcal{E}^n = (H, \{\rightsquigarrow_i\}_{i\in Ag}, V)$

An element $(w, e_1, \ldots, e_n)$ of $\mathcal{M}\mathcal{E}^*$ is seen as a history $we_1 \ldots e_n$, and $we_1 \ldots e_n \rightsquigarrow_i w'e'_1 \ldots e'_n$ if $w\, R_i\, w'$ and $e_k\, \mathsf{R}_i\, e'_k$ for all k.

Plan

Definition

Regular structures

A relational structure $\mathcal{S} = (D, \{\rightsquigarrow_i\}_{i \in Ag}, V)$ is regular if:

- $D \subseteq \Sigma^*$ is a regular language over some alphabet Σ,
- for each $i \in Ag$, $\rightsquigarrow_i$ is a regular relation,
- for each $p \in AP$, $V(p) \subseteq D \subseteq \Sigma^*$ is a regular language.

In other words, the structure is representable by finite automata.

Regular relations

Regular relations

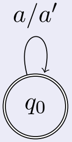
A binary relation over words is regular if it is recognized by a synchronous transducer.

Example: synchronous perfect recall

- Let $\rightsquigarrow \subseteq \Sigma \times \Sigma$ be an accessibility relation.
- Extend it to words:

$$a_1 \dots a_n \rightsquigarrow a'_1 \dots a'_n \text{ if } a_k \rightsquigarrow a'_k \text{ for each } k.$$

- Recognized by:



if $a \rightsquigarrow a'$

From DEL to automata

Theorem: from DEL to automata

For every epistemic model $\mathcal{M}$ and propositional event model $\mathcal{E}$, $\mathcal{M}\mathcal{E}^*$ is an automatic structure, and we can build recognizers.

Define $\mathcal{A} = (\Sigma, Q, \delta, q_\iota, F)$, where

- $\Sigma = W \cup \mathsf{E}$,
- $F = \{q_\nu \mid \nu \subseteq AP\}$, $Q = F \uplus \{q_\iota\}$, and $\forall w \in W$, $\forall e \in \mathsf{E}$,

$$\delta(q_\iota, w) = q_{\nu(w)} \quad \delta(q_\iota, e) \text{ is undefined,} \quad \delta(q_\nu, w) \text{ is undefined}$$

$$\delta(q_\nu, e) = \begin{cases} q_{\nu'}, \text{ with } \nu' = \{p \mid \nu \models \mathsf{post}(e)(p)\} & \text{if } \nu \models \mathsf{pre}(e) \\ \text{undefined} & \text{otherwise.} \end{cases}$$

Fact

$\mathcal{A}$ accepts exactly the histories in $\mathcal{M}\mathcal{E}^*$.

Plan

1. Dynamic Epistemic Logic (DEL)
2. From DEL to regular structures
3. Epistemic planning and epistemic protocol synthesis

Epistemic planning

The epistemic planning problem (EPP)

Input:

- a pointed initial epistemic model $(\mathcal{M}, w)$
- an event model $\mathcal{E}$
- a goal formula $\varphi \in \mathcal{L}^{EL}$

Output:

- Is there $e_1 \dots e_n$ s.t. $(\mathcal{M}, w) \otimes (\mathcal{E}, e_1) \otimes \dots \otimes (\mathcal{E}, e_n) \models \varphi$?

Example

Is there a finite sequence of events such that in the end, Alice and Bob both know how is the coin?

Epistemic planning

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- Is there $e_1 \dots e_n$ s.t. $\mathcal{M}\mathcal{E}^*, we_1 \dots e_n \models \varphi$?

Example

Is there a finite sequence of events such that in the end, Alice and Bob both know how is the coin?

Epistemic planning

The propositional epistemic planning problem (propositional EPP)

Input:

- a pointed initial epistemic model $(\mathcal{M}, w)$
- a propositional event model $\mathcal{E}$
- a goal formula $\varphi \in \mathcal{L}^{EL}$

Output:

- Is there $e_1 \ldots e_n$ s.t. $\mathcal{M}\mathcal{E}^*, we_1 \ldots e_n \models \varphi$?

Example

Is there a finite sequence of events such that in the end, Alice and Bob both know how is the coin?

Theorem [Yu et al. 2013]

The propositional epistemic planning problem is decidable.

Our contribution

Theorem [Yu et al. 2013]

The propositional epistemic planning problem is decidable.

[Yu et al. 2013] prove that a finite search tree is sufficient.

We propose:

Alternative proof, based on automata techniques.

- Provides better upper bounds on the complexity.
- Builds an automaton that generates all the solution plans.
- Our approach allows to solve a much more general problem.

Uniform strategies

Use techniques developed for computing uniform strategies with rational relations.

Uniform strategy problem (roughly):

Input:

- A finite game arena
- n transducers that recognize relations between plays
- A CTL*K$_n$ formula φ

Output:

- Is there a strategy for Player 1 that verifies φ?

[M., Pinchinat 2013]

The uniform strategy problem is decidable (nonelementary).

Back to epistemic planning

Take an instance $((\mathcal{M}, w), \mathcal{E}, \varphi)$ of the propositional EPP.

- Build an automatic representation $(\mathcal{A}, \{T_i\}_{i \in Ag})$ of $\mathcal{M}\mathcal{E}^*$
- See $\mathcal{A}$ as a one player game arena
- Look for a strategy (a play) that verifies $\mathbf{EF}\varphi$ (or $\mathbf{AF}\varphi$)

Because the uniform strategy problem is decidable for regular relations, we get the decidability of Propositional EPP, and more:

Theorem

Arbitrary relations: Prop. EPP is in $(d(\varphi) + 1)$-EXPTIME
Equivalence relations: Prop. EPP is in $(ad(\varphi) + 1)$-EXPTIME
Synthesis: Build automaton that generates all the solution plans.

Epistemic protocol synthesis in DEL

Epistemic planning

- finite sequence of events
- reach epistemic objective

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Epistemic protocol synthesis

- infinite tree of events
- $\mathrm{CTL}^*\mathrm{K}_n$ specification

Epistemic protocol synthesis in DEL

Epistemic planning

- finite sequence of events
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Epistemic protocol synthesis

- infinite tree of events
- $\mathrm{CTL}^*\mathrm{K}_n$ specification

Theorem

The propositional epistemic protocol synthesis problem is decidable.

Same techniques and same complexity as for epistemic planning.

Conclusion

- Connection between DEL-generated structures and regular structures
- This bridge allows us to apply existing automata techniques
- Alternative decidability proof for Propositional EPP
- Side results:
  - Improved complexity upper-bounds
  - Synthesize an automaton that recognizes the solution plans
- Same techniques apply to solve the generalized problem of Epistemic protocol synthesis

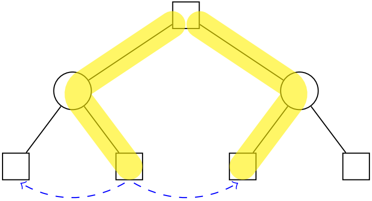
Future work

- Are these decision procedures optimal?
- Consider asynchronous variants of DEL
- Put strategic aspects in DEL.
  - By whom is an event triggered?
  - What are the agents' objectives?

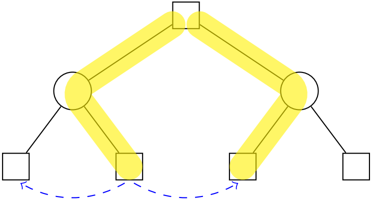
Thank you!

Semantics of $\boxed{\rightsquigarrow}$ and $\boxed{\boxed{\rightsquigarrow}}$

Blue arrows: $\rightsquigarrow$



$\boxed{\rightsquigarrow}$: Strict quantifier



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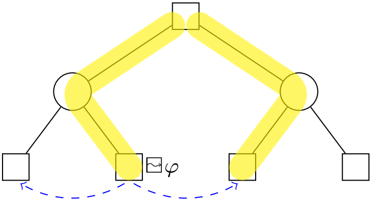

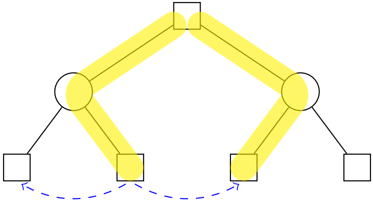

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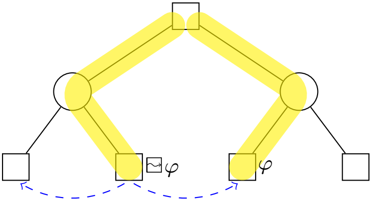

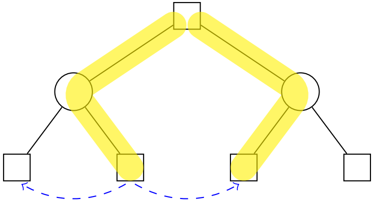

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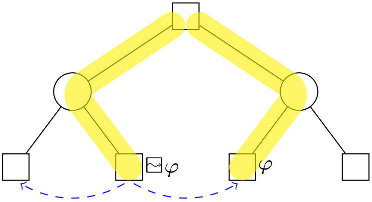

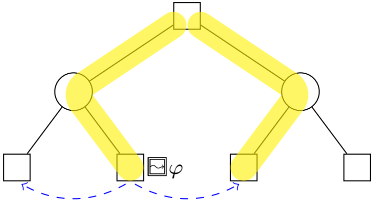

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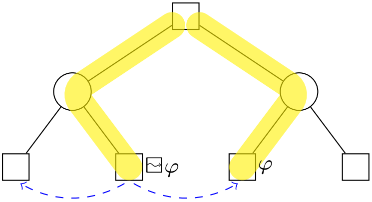

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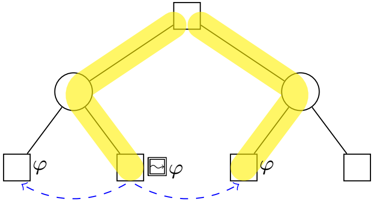

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