Concurrent Game Structures with Roles

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Exploring notion of homogeneity in strategic situations.

Exploit the fact that in many real-world situations multiple agents play the same role.

Jamroga & Dix:
“Do agents make model checking explode (computationally)?”
\langle \{a, b\} \rangle \diamond p
\[\langle \alpha_1, \ldots, \alpha_n \rangle\]
\[ \langle \alpha_1, \ldots, \alpha_n \rangle \rightarrow \langle (n_1, \ldots, n_k), \ldots, (m_1, \ldots, m_l) \rangle \]
Definition (cgs)

- $\mathbb{A} : Q \times \mathcal{A} \rightarrow \mathbb{N}^+$ is the number of available actions in a given state for a given agent.
- $\delta : Q \times \bigcup_{q \in Q} \mathcal{A}(q) \rightarrow Q$, defines a successor state for any state and a complete profile for $q$. 
Definition (RCGS)

- $R$ is a non-empty set of roles.
- $\mathcal{R} : Q \times A \rightarrow R$ denotes the role the given agent belongs to in the given role.
- $\mathcal{A} : Q \times R \rightarrow \mathbb{N}^+$ is the number of available actions in a given state for a given role.
- $\delta : Q \times \bigcup_{q \in Q} P(q) \rightarrow Q$, defines a successor state for any state and a complete (voting) profile for $q$. 
Definition (Complete profile)

- A complete vote for a role $r$ in $q$

$$(n_1, \ldots, n_k)$$
Definition (Complete profile)

- A complete vote for a role $r$ in $q$
  \[(n_1, \ldots, n_k)\]

- A complete voting profile for $q$
  \[\langle (n_1, \ldots, n_k), \ldots, (m_1, \ldots, m_l) \rangle\]
For cgs there are $2^n$ possible complete profiles.

For rcgs there are $n$ possible complete profiles.

**Figure:** Simple 1-tier sensor network
Figure: Simple 1-tier sensor network

- For $\text{CGS}$ there are $2^n$ possible complete profiles.
- For $\text{RCGS}$ there are $n$ possible complete profiles.
Figure: Simple 2-tier sensor network.
Size of models

Number of ways $r$ agents can choose a number $\{1, \ldots, \alpha\}$, is

\[
\binom{r + \alpha - 1}{\alpha} = \frac{(r + \alpha - 1)!}{r!(\alpha - 1)!}
\]
Size of models

The number of out-edges at $q$ is

$$\prod_{r \in R} \frac{(|R(q, r)| + (A(q, r) - 1))!}{|R(q, r)|!(A(q, r) - 1))!}$$

(1)
Size of models

The number of out-edges at $q$ is

$$\prod_{r \in R} \frac{(|\mathcal{R}(q, r)| + (A(q, r) - 1))!}{|\mathcal{R}(q, r)|!(A(q, r) - 1))!}$$ (1)

Observe that

$$\frac{(r+(a-1))!}{r!(a-1)!} \leq a^r \quad \text{and} \quad \frac{(r+(a-1))!}{r!(a-1)!} \leq r^a$$ (2)
Size of models

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Observe that

$$\frac{(r+(a-1))!}{r!(a-1)!} \leq a^r \quad \text{and} \quad \frac{(r+(a-1))!}{r!(a-1)!} \leq r^a$$  \hspace{1cm} (2)

This gives that the size of the model is bounded by both

$$\mathcal{O}\left( \sum_{q \in Q} \prod_{r \in R} |\mathcal{R}(q, r)|^{\mathcal{A}(q, r)} \right)$$  \hspace{1cm} (3)

and

$$\mathcal{O}\left( \sum_{q \in Q} \prod_{r \in R} \mathcal{A}(q, r)^{|\mathcal{R}(q, r)|} \right)$$  \hspace{1cm} (4)
Theorem

For a RCGS $S$ and a formula $\phi$, $m\text{check}(S, \phi)$ takes time $\mathcal{O}(|\phi| \times e^2)$, where $e$ is the total number of transitions in $S$. 
**Theorem**

*For a RCGS $S$ and a formula $\phi$, $mcheck(S, \phi)$ takes time $\mathcal{O}(|\phi| \times e^2)$, where $e$ is the total number of transitions in $S$.*

**Theorem**

*Given any CGS-model $M$, we have, for all $S \in f^-(M)$, that the complexity of running $mcheck(S, \phi)$ is $\mathcal{O}(mcheck(M, \phi))$.***
We introduce roles to explore different levels of homogeneity between agents.

In cases where agents have some level of homogeneity we find that the $\text{RCGS}$ model is smaller than the $\text{CGS}$ model.

Equivalent wrt. $\text{ATL}$. 