Uniform strategies
Strategic Reasoning 2013

Bastien Maubert  Sophie Pinchinat\textsuperscript{1}  Laura Bozzelli\textsuperscript{2}

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\textsuperscript{1}IRISA, Rennes, France
\textsuperscript{2}UPM, Madrid, Spain
1 Motivation

2 Uniform strategies

3 Synthesizing fully-uniform strategies
Two distinct notions of uniform strategies in the literature:

- In games with imperfect information:
  - Uniform strategies [Van Benthem, 2001]
  - Observation-based strategies [Chatterjee et al., 2006]

- In model-checking games for logics of imperfect information:
  - Uniform strategies [Väänänen, 2007]
  - Coherent strategies [Grädel, 2012]

**Common point**

They are different notions of uniformity, but both represent constraints on strategies (they limit the set of allowed strategies), and both constraints involve sets of plays.
Motivation
Uniform strategies
Synthesizing fully-uniform strategies

Games with imperfect information

**Arena**
- Player 1 has a partial observation of plays
- Observational equivalence relation on finite plays: \( \rho \sim \rho' \)

**Strategies**
- Player 1 cannot use information she does not have
- Strategies must be *observation-based*, or *uniform*:
  \[ \rho \sim \rho' \Rightarrow \sigma(\rho) = \sigma(\rho') \]

Constraint on sets of equivalent plays
Games with imperfect information

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Picture

- Imperfect-information games
- Games for imperfect-information logics
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Uniform strategies

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Uniform strategies

Games for non-interference
Imperfect-information games
Games with epistemic winning condition

Games for diagnosis
Games for imperfect-information logics
Two-player turn-based game arenas

\[ \text{Prop} = \{ p, q, r, \ldots \} \]

\[ G = (V, E, v_I, \ell) \text{ where} \]
- \( V = V_1 \uplus V_2 \) : positions
- \( E \subseteq V \times V \) : edges
- \( v_I \in V \) : initial position
- \( \ell : V \rightarrow 2^{\text{Prop}} \) : valuation

- \( \pi \in \text{Plays}_\omega \)
- \( \pi[0..i] = \pi[0]\pi[1] \ldots \pi[i] \)
- Strategy (for Player 1):
  \( \sigma : V^*V_1 \rightarrow V \)
- \( \text{Out}(\sigma) \subseteq \text{Plays}_\omega \)
Two-player turn-based game arenas

\[
Prop = \{p, q, r, \ldots\}
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\[
G = (V, E, v_I, \ell) \text{ where}
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\begin{itemize}
  \item \(V = V_1 \uplus V_2\) : positions
  \item \(E \subseteq V \times V\) : edges
  \item \(v_I \in V\) : initial position
  \item \(\ell : V \rightarrow 2^{Prop}\) : valuation
\end{itemize}

\[
\pi \in Plays_\omega
\]

\[
\pi[0..i] = \pi[0]\pi[1] \ldots \pi[i]
\]

Strategy (for Player 1):
\[
\sigma : V^*V_1 \rightarrow V
\]

\[
Out(\sigma) \subseteq Plays_\omega
\]
Specification language: $\mathbb{RLTL}$

- LTL for temporal properties
- $\mathbb{R}$ for grabbing sets of plays

**Syntax**

$$\mathbb{RLTL} : \quad \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \bigcirc \varphi \mid \varphi U \varphi \mid R \varphi$$

$R \varphi$ means: $\varphi$ holds in all *related* plays.
Examples of formulas

Imperfect-information strategies

Semantics of $\mathcal{R}$: Player 1’s observation

$$\mathbf{G}(p_1 \rightarrow \bigvee_{m \in \text{Move}} \mathbf{R} \circ m)$$

”Whenever it is Player 1’s turn, indistinguishable plays will be extended with the same move $m$”

Games with opacity condition

- Some positions denote a “secret” information
- Player 1 doesn’t want Player 2 to find it
- Semantics of $\mathcal{R}$: Player 2’s observation

$$\mathbf{G} \neg \mathbf{R}_\text{secret}$$
Semantics

- $\Pi \subseteq \text{Plays}_\omega$: the universe
- $\sim \subseteq \text{Plays}^2_\ast$: any binary relation on finite plays.
  - Knowledge
  - Beliefs
  - Other relations, not related to any kind of “information”

Take $\pi \in \Pi$, $i \in \mathbb{N}$.

$\Pi, \pi, i \models \varphi$

$p, \neg \varphi, \varphi \land \psi \ldots$
$\Pi, \pi, i \models \bigcirc \varphi$ \quad \text{if} \quad $\Pi, \pi, i + 1 \models \varphi$
$\Pi, \pi, i \models \varphi \bigcup \psi \ldots$

$\Pi, \pi, i \models \text{R} \varphi$ \quad \text{if} \quad \text{for all } \pi' \in \Pi, j \in \mathbb{N}, \text{ such that } \pi[0..i] \sim \pi'[0..j], \Pi, \pi', j \models \varphi$
Two possible universes

Yellow lines: $\sigma$

$\Pi = \text{Out}(\sigma)$: Strict uniformity

$\Pi = \text{Plays}_\omega$: Full uniformity
Two possible universes

Yellow lines: $\sigma$

$\Pi = \text{Out}(\sigma)$: Strict uniformity

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Two possible universes

Yellow lines: $\sigma$

\[ \Pi = \text{Out}(\sigma) : \text{Strict uniformity} \quad \text{and} \quad \Pi = \text{Plays}_\omega : \text{Full uniformity} \]
Two possible universes

Yellow lines: $\sigma$

$\Pi = \text{Out}(\sigma)$: Strict uniformity

$\Pi = \text{Plays}_\omega$: Full uniformity
Two notions of uniform strategies

As strategy $\sigma$ is $\varphi$-strictly uniform if:
for all $\pi$ in $\text{Out}(\sigma)$,

$$\text{Out}(\sigma), \pi, 0 \models \varphi$$

A strategy $\sigma$ is $\varphi$-fully uniform if:
for all $\pi$ in $\text{Out}(\sigma)$,

$$\text{Plays}_\omega, \pi, 0 \models \varphi$$
Utilisations of strictly and fully

- Observation based strategies: strict uniformity.
- Coherent strategies: strict uniformity.
- Games with epistemic winning condition (LTLK)
  - Agent knows the strategy: Strict uniformity.
  - Agent ignores the strategy: Full uniformity.
  Example: study strategies of Player 1, while the objective concerns the knowledge of Player 2.

**Question:**

How to decide the existence of a uniform strategy?
Synthesizing fully-uniform strategies

The fully-uniform strategy problem (FUS)

Input: Finite arena $G$, relation $\rightsquigarrow$, $\varphi \in \mathbb{RLTL}$.
Output: Yes if there is a $\varphi$-fully uniform strategy in $G$.

How to finitely represent a binary relation?
Motivation

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The fully-uniform strategy problem (FUS)

Input: Finite arena $G$, relation $\sim$, $\varphi \in \mathbb{IRLTL}$.
Output: Yes if there is a $\varphi$-fully uniform strategy in $G$.

How to finitely represent a binary relation?

⇒ Finite state transducers (FST)
They recognize rational relations.
Finite state transducers and rational relations

Relation recognized by this transducer:

\[ w \sim w' \text{ if } |w|_a = |w'|_a \]
FUS with rational relations

**FUS$_{rat}$**

Input:
- Finite arena $G$
- FST $T$ representing a rational relation $\sim$
- $\varphi \in \text{RLTL}$

Output: Yes if there is a $\varphi$-fully uniform strategy in $G$.

**Theorem**

FUS$_{rat}$ is decidable
Reduction to LTL games

Let $G, T, \varphi$ be an instance of $\text{FUS}_{rat}$ ($T$ recognizes $\rightsquigarrow$).

$\mathbb{R}$ depth

$d_{\mathbb{R}}(\varphi)$ is the maximum nesting of $\mathbb{R}$ modalities in $\varphi$.

Reduction to LTL games

Iterate $d_{\mathbb{R}}(\varphi)$ times:

- Powerset construction from $G$ and $T$
- If $\psi \in \text{LTL}$, $\mathbb{R}\psi$ can be evaluated positionally
- $\Rightarrow$ Elimination of innermost $\mathbb{R}\psi$ subformulas of $\varphi$

Solve the LTL game, synthesize a finite-memory strategy.
Complexity

Solving LTL games:

$2\text{EXPTIME}$-complete [Pnueli & Rosner, 85]

A non-elementary decision procedure, essentially optimal.

FUS$_{\text{rat}}$ is non-elementary-complete

More precisely, if we assume that $d_R(\varphi) \leq d$, then it is

- $2\text{EXPTIME}$-complete if $d \leq 2$
- $d\text{EXPTIME}$-complete if $d > 2$

Lower bounds: encoding of exp[$d$]-space bounded alternating Turing Machines.
Conclusion

- Two general notions of uniform strategies
- Subsumes both previous notions of uniform strategies
- A language (\(\text{RLTL}\)) to specify uniformity constraints
- Enables to represent constraints on strategies, or winning conditions with epistemic or belief features
- The fully-uniform strategy problem is decidable, for a very general class of relations.
Current and future work

- Additional assumptions on $\sim$ that make FUS elementary
- Decidability status of the strictly-uniform strategy problem, for various classes of relations
- $n$ agents
- Language allowing both strict and full semantics: $R_s$ and $R_f$
- Add ATL-like modalities?
- What if we take $\mu$-calculus instead of LTL?
Thank you

Questions?
Games with Imperfect Information

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\[ \pi[0, i] \sim \pi'[0, j] \text{ if } \pi[0, i] \text{ and } \pi'[0, j] \text{ are observationally equivalent} \]

\[ \varphi = \mathbf{G}(p_1 \rightarrow \bigvee_{a \in \text{Act}} R \circ p_a) \]

**Theorem**

A strategy \( \sigma \) for Player 1 is “uniform” iff \( \sigma \) is \( \varphi \)-strictly uniform.
Dependence Logic

\[ \pi[0, i] \sim \pi'[0, j] \text{ if } \pi[i] = (\text{dep}(t_1, \ldots, t_n), s), \]
\[ \pi'[j] = (\text{dep}(t_1, \ldots, t_n), s') \text{ and } s \text{ and } s' \text{ agree on } t_1, \ldots, t_{n-1}. \]
\[ \varphi = \mathcal{G}(d \to \bigvee_{a \in \text{Dom}} \mathcal{R}(t_n = a)) \]

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A strategy \( \sigma \) is “uniform” iff it is \( \varphi \)-strictly uniform.
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**Dependence Logic**

\[ \forall x_0 \forall x_1 \varphi' \]

\[ \forall x_1 \varphi' \]

\[ \pi[0, i] \sim \pi'[0, j] \text{ if } \pi[i] = (\text{dep}(t_1, \ldots, t_n), s), \]

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\[ \varphi = G(d \to \bigvee_{a \in \text{Dom}} R(t_n = a)) \]

**Theorem**

A strategy \( \sigma \) is “uniform” iff it is \( \varphi \)-strictly uniform...
Dependence Logic

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\pi[0, i] \sim \pi'[0, j] \text{ if } \pi[i] = (\text{dep}(t_1, \ldots, t_n), s), \quad \pi'[j] = (\text{dep}(t_1, \ldots, t_n), s') \text{ and } s \text{ and } s' \text{ agree on } t_1, \ldots, t_{n-1}.
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\varphi = G\left(d \rightarrow \bigvee_{a \in \text{Dom}} R(t_n = a)\right)
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Theorem

A strategy \(\sigma\) is “uniform” iff it is \(\varphi\)-strictly uniform
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Dependence Logic

Theorem
A strategy $\sigma$ is "uniform" iff it is $\varphi$-strictly uniform

$$\varphi = G(d \rightarrow \bigvee_{a \in Dom} R(t_n = a))$$
When is full uniformity needed?

- $R$: Player 2’s knowledge.
- Player 1: perfect information.
- $S \subseteq V$: a secret property.
- Player 1 wants to prevent Player 2 from knowing $S$

Let $\varphi = G \neg \mathbf{R} S$.

**Opacity guarantee**

Player 1 can protect the secret if she has a $\varphi$-fully uniform strategy.

Player 2 can consider possible plays that are not induced by Player 1’s strategy.
Powerset construction

- $T$: transducer that recognizes $\sim$
- “Product” of the arena $\mathcal{G}$ with $T$
- We simulate the *nondeterministic* execution of $T$ along the game
- $T$ reads the current play and nondeterministically outputs related plays
Powerset construction

- What we want: information sets, to evaluate $\mathbb{R}_\varphi$ positionally. Information set = set of last positions of equivalent plays
- What we need to remember:
  - For the execution of $T$: The set $S$ of states in which the transducer may be (nondeterminism)
  - For computing $I$: For each $q \in S$, for each possible execution of $T$ that ends in $q$, remember the last letter/position on the output. It is the set $\text{Last}(q)$.

A position: $(v, S, \text{Last})$

Computing $I$:

$$I(v, S, \text{Last}) = \bigcup_{q \in S \cap Q_F} \text{Last}(q)$$
Idea of the $\mathcal{R}\varphi$ elimination

$\varphi \in \text{LTL}$

**Positionality**

- Whether $\pi, i \models \mathcal{R}\varphi$ or not does not depend on $\pi[i + 1, \ldots]$ (quantification over plays).
- $\pi[0 \ldots i]$ determines the set of equivalent plays.
- In the powerset construction, the relevant information on the past is in the position $\Rightarrow$ For full uniformity the semantics of $\mathcal{R}\varphi$ can be defined positionally:

**Positional semantics**

$$(v, S, Last) \models \mathcal{R}\varphi \text{ if } v' \models A\varphi \text{ for all } v' \in I(v, S, Last)$$
Idea of the $R_\varphi$ elimination

Take $\mathcal{G}, \sim, \varphi$ an instance. Let $\mathcal{G}$ be the powerset construction.

**Marking**

- Let $R_{\varphi_1}, \ldots, R_{\varphi_n}$ be all the subformula of $\varphi$ such that $\varphi_i \in \text{LTL}$
- For each $\hat{v} \in \hat{V}$, for all $i$, if $\hat{v} \models R_{\varphi_i}$, let $p_{R_{\varphi_i}} \in \ell(\hat{v})$
- $\hat{\varphi} := \varphi[ p_{R_{\varphi_1}} / R_{\varphi_1}, \ldots, p_{R_{\varphi_n}} / R_{\varphi_n}]$
- We have $d(\hat{\varphi}) = d(\varphi) - 1$

Lift $T$ to $\mathcal{G}$: $\hat{T}$

Player 1 has a $\varphi$-fully uniform strategy in $\mathcal{G}$ with $[T]$ iff she has a $\hat{\varphi}$-fully uniform strategy in $\mathcal{G}$ with $[\hat{T}]$