Restricted Manipulation in Iterative Voting: Convergence and Condorcet Efficiency

Umberto Grandi

Department of Mathematics
University of Padova

16 March 2013

Joint work with Andrea Loreggia, Francesca Rossi, K. Brent Venable and Toby Walsh
Manipulation in elections is usually considered a bad thing, to be avoided or at least to be made computationally difficult to achieve.

What if we can get a better outcome with iterated manipulation of simple rules, rather than complex-information-costly-almost-strategy-proof rules?
In practice, *iterative manipulation* do occur:

**Iterative response to repeated polls**

**Approval voting with iterative manipulation**

*Image source: Wikipedia, Doodle.com*
Outline

1. The setting
   - Strategic manipulation and voting rules
   - Voting games / iterative voting
   - Restricted manipulation: $M1$ and $M2$

2. Theoretical and experimental evaluation
   - Convergence: Yes! (unknown for STV)
   - Axiomatic properties: transfer to iterative rules
   - Condorcet efficiency and average position of the winner: Increase!
Voting Rules

What is a voting rule:

- We start from a set of individuals $1, \ldots, n$ expressing preferences over candidates $\{c_1, \ldots, c_m\}$: a profile $(<_1, \ldots, <_n)$ of linear orders
- The voting rule uses this information to compute a (set of) winner(s).

Examples of voting rules:

- **Positional Scoring Rules** give $s_j$ points to candidates in position $j$ in individual preferences, and elect the candidates with maximal score. We consider: Plurality, Borda, 2-approval, 3-approval, veto.
- **Copeland, Maximin** ...
- **Single Transferable Vote** deletes the candidate with the least first positions in individual preferences, transfer the votes to the succeeding candidate, and iterates until there is one candidate which has the majority of first positions.

Assumption: linear tie-breaking (for these slides $a > b > c > \ldots$)
Strategic Manipulation

Manipulation occurs whenever a voter changes her ballot in her favour:

\[
\begin{align*}
    a &\succ b &\succ c \\
    b &\succ c &\succ a \\
    c &\succ b &\succ a \\
\end{align*}
\]

Plurality: \(a\)

\[
\begin{align*}
    a &\succ b &\succ c \\
    b &\succ c &\succ a \\
\end{align*}
\]

Plurality: \(b\)
Strategic Manipulation

Manipulation occurs whenever a voter changes her ballot in her favour:

\[
\begin{align*}
& a \succ b \succ c \\
& b \succ c \succ a \\
& c \succ b \succ a \\
\end{align*}
\]

Plurality: \( a \)

\[
\begin{align*}
& a \succ b \succ c \\
& b \succ c \succ a \\
& b \succ c \succ a \\
\end{align*}
\]

Plurality: \( b \)

Is there any chance to avoid manipulation?

**Theorem [Gibbard-Satterthwaite]**

*Given a voting rule \( F \), one of the following facts must be true: (i) there is a candidate that never wins (ii) \( F \) is a dictatorship, (iii) \( F \) can be manipulated.*

Needless to say, all voting rules presented are manipulable...

M. A. Satterthwaite, Strategy-proofness and Arrows conditions... *JET*, 1975.
Strategic manipulation in elections defines a voting game:

- Strategies are linear orders: individuals can change their preferences to obtain a better outcome
- The outcome is the result of the voting rule
- Utilities are defined by the truthful preferences of individuals

R. Meir Et Al. Convergence to equilibria in plurality voting. AAAI-2010.
Voting Games / Iterative Voting

Strategic manipulation in elections defines a voting game:

- Strategies are linear orders: individuals can change their preferences to obtain a better outcome
- The outcome is the result of the voting rule
- Utilities are defined by the truthful preferences of individuals

Definition

Given a set of manipulation moves \( M \), a voting rule \( F \) (and a turn function) the iterated voting rule \( F^M \) associates with every profile \( \mathbf{b} \) the outcome of convergent iteration of manipulation moves in \( M \) (or \( \uparrow \) if it does not converge).

Unrestricted manipulation does not always converge! But if it does, it converges to a Nash equilibrium of the voting game associated to \( F \).

R. Meir Et Al. Convergence to equilibria in plurality voting. AAAI-2010.
Restricted Manipulation

Manipulation moves studied in the literature:

- **Best response** (no restriction): choose the ballot that changes the outcome of the election in the best way.
- **k-pragmatist**: put in first position your favourite candidate among the top $k$ in the outcome of the voting rule.

Manipulation moves studied in the literature:

- **Best response** (no restriction): choose the ballot that changes the outcome of the election in the best way.
- **k-pragmatist**: put in first position your favourite candidate among the top \( k \) in the outcome of the voting rule.

How to **evaluate** a manipulation move?

<table>
<thead>
<tr>
<th>Convergence</th>
<th>Computation</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guaranteed</td>
<td>Not costly</td>
<td>Low</td>
</tr>
<tr>
<td>(small number of steps)</td>
<td>(not NP-hard!)</td>
<td>(top candidate, scores..)</td>
</tr>
</tbody>
</table>
Restricted Manipulation: $M1$

Iteration starts at $b^0$ (truthful) and continues to $b^1, \ldots, b^k$ until convergence.

**M1**

*Move to the top the second-best candidate in $b^0_i$ (truthful), unless the current winner $w = F(b^k)$ is already her best or second-best candidate in $b^0_i$ (truthful).*

\[
\begin{align*}
\text{Plurality: } a &\succ b \succ c \succ d \\
\text{Plurality: } a &\succ b \succ c \succ d \\
\text{Plurality: } b &\succ d \succ c \succ a
\end{align*}
\]

Minimal computation cost, minimal information required.

A side note: $b$ is the **Condorcet winner**.
Restricted Manipulation: $M2$

**M2**

move to the top the best candidate in $b^0_i$ (truthful) which is above $w = F(b^k_i)$ in $b^k_i$ (reported), among those that can become the new winner of the election

\[
\begin{align*}
a & \succ b \succ c \succ d \\
b & \succ c \succ a \succ d \\
d & \succ a \succ b \succ c \\
c & \succ d \succ b \succ a \\
\text{Plurality: } a
\end{align*}
\]

\[
\begin{align*}
a & \succ b \succ c \succ d \\
c & \succ b \succ a \succ d \\
d & \succ a \succ b \succ c \\
c & \succ d \succ b \succ a \\
\text{Plurality: } c
\end{align*}
\]

\[
\begin{align*}
a & \succ b \succ c \succ d \\
c & \succ b \succ a \succ d \\
a & \succ d \succ b \succ c \\
c & \succ d \succ b \succ a \\
\text{Plurality: } a
\end{align*}
\]

Low computation cost, low information required (score, majority graph).
Convergence and Axiomatic Properties

**Theorem**

*Restricted iterated voting with M1 converges for every voting rule.*

Proof idea: M1 can be applied only once by each individual.

**Theorem**

*Restricted iterated voting with M2 converges for PSR, Copeland and Maximin.*

Proof idea: the score of the winner increases at every step, or remains the same and the candidate moves up in the tie-breaking order.

Axiomatic properties are **preserved** at every step of the iteration:

Example: if $F$ is unanimous (elects a candidate if it is on top of every individual preference) then $F^{M1}$ and $F^{M2}$ are unanimous.
Condorcet Efficiency

For Plurality better 2P and 3P, for all others $M2$ is better. Positive performance of $M1$, even if little changes.
Motivational Intermezzo ($M2$)

One further motivation for iterated manipulation is that the Condorcet winner may be extracted without having to ask for the full profile.

But: is it more costly to iterate or to ask for the full profile?

<table>
<thead>
<tr>
<th>Method</th>
<th># profiles</th>
<th>average # steps</th>
<th>maximal # steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality</td>
<td>2902</td>
<td>11.8</td>
<td>27</td>
</tr>
<tr>
<td>STV</td>
<td>1173</td>
<td>1.7</td>
<td>7</td>
</tr>
<tr>
<td>Borda</td>
<td>1961</td>
<td>8.1</td>
<td>31</td>
</tr>
<tr>
<td>2-Approval</td>
<td>2395</td>
<td>9.1</td>
<td>17</td>
</tr>
</tbody>
</table>

Profiles are $50 \times 5$, maximal number of iterations is 27: good for Plurality!
Iteration takes place between 10% and 30% of the cases:
Not very costly, given the increase in Condorcet efficiency!
Average Position of the Winner

How much preferred is the winner in average?

For all voting rules (except for Borda) the position of the winner increases
Conclusions and Future Work

We introduced two new restricted manipulation moves which are easy to compute and need small amount of information, and we evaluated:

- Convergence of restricted iterative voting
- Condorcet efficiency
- Average position of the winner
- Number of iteration steps

Restricted manipulation in iterative voting increases the Condorcet efficiency and the average position of the winner in a limited number of steps.

Lots of future questions:

- What is the best restricted manipulation move?
- Other parameters to judge iterated rules?
- More realistic distribution of preferences (no IC assumption).

Thank you for your attention!
Condorcet Efficiency

Non-Iterative version

M1

M2

2-pragmatists

3-pragmatists
Higher efficiency for $n = 20$, stabilizes at around $n = 50$. 