Reasoning about Strategies under Partial Observability and Fairness Constraints

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Running Example: A simple card game [1]

Three cards: A, K, Q
(A wins over K, K over Q, Q over A);

A player, a dealer.

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the player can change his card with the one on table.

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The dealer gives a card and keeps one;
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Variant: the player can play infinitely.

Running Example: A simple card game

A, K → A, Q
K, A → K, Q
Q, A → Q, K
A, K → A, Q
K, A → K, Q
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Reasoning about strategies

Model checking problem:

does the player have a strategy to win?
Reasoning about strategies

Model checking problem:  
**does the player have a strategy to win?**

⇒ it depends on the semantics!
Reasoning about strategies

Model checking problem:

*does the player have a strategy to win?*

Under *ATL*, we consider all strategies. The player has a strategy to win, even if he cannot play it:
e.g., in \( \langle A, K \rangle \), keep the card; in \( \langle A, Q \rangle \), exchange it.
Reasoning about strategies

Model checking problem:
does the player have a strategy to win?

\textit{ATL}: yes.

Under \textit{ATL}_{ir}, we consider only memoryless uniform strategies. There is no uniform strategy to win, because the player cannot distinguish, e.g., \( \langle A, K \rangle \) and \( \langle A, Q \rangle \), (winning actions are different in each case).
Reasoning about strategies

Model checking problem:
\textbf{does the player have a strategy to win?}

\textit{ATL}: yes.

\textit{ATL}_{ir}: no.

If we consider \textit{ATL}_{ir} with a \textbf{fair dealer} and an \textbf{infinite play}, the player can eventually win: just use one uniform strategy, the right pair will finally come.
Reasoning about strategies

Model checking problem:

\textbf{does the player have a strategy to win?}

\textit{ATL}: yes.

\textit{ATL}_{ir}: no.

\textit{ATL}_{ir} + fair dealer and infinite play: yes.

\Rightarrow \textit{ATL}^{F}_{po}: branching time, knowledge, memoryless uniform strategies and unconditional fairness constraints.
Outline

Strategies, Temporal Logics and Fairness

Strategies under Partial Observability and Fairness Constraints

Conclusion and Perspectives
**ATL**, reasoning about **strategies** of the agents. [2]

**Syntax:** Strategic modalities: \(⟨\Gamma⟩ \mathbf{X} \phi, [\Gamma] \mathbf{G} \phi, ⟨\Gamma⟩[\phi_1 \mathbf{U} \phi_2] \), etc.

**Semantics:** A state \(s\) satisfies \(⟨\Gamma⟩ \pi\) iff there exists a set of **strategies** for agents in \(\Gamma\) such that all enforced paths satisfy \(\pi\).

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**Semantics:** A state $s$ satisfies $⟨\Gamma⟩\pi$ iff there exists a set of **strategies** for agents in $\Gamma$ such that all enforced paths satisfy $\pi$.

**Model checking:**

$$\llbracket [\Gamma]G \phi \rrbracket = \nu Z.\llbracket \phi \rrbracket \cap Pre[\Gamma](Z)$$

where $Pre[\Gamma](Z)$ is the set of states from which $\Gamma$ cannot avoid to reach $Z$ in one step.

**ATL**$_{ir}$, memoryless uniform strategies [3]

Only **memoryless uniform** strategies:

\[ f_a : S \rightarrow \text{Act} \text{ such that } s \sim_a s' \implies f_a(s) = f_a(s') \]

**Semantics:** A state \( s \) satisfies \( \langle \Gamma \rangle \pi \) iff there exists a set of **memoryless uniform** strategies for agents in \( \Gamma \) such that all paths enforced **from all** \( s' \sim_\Gamma s \) satisfy \( \pi \).

FairCTL: time and fairness constraints [4]

Add a set of **fairness constraints** $FC \subseteq 2^S$ to the model; 
⇒ unconditional state-based fairness.

Only **fair paths** are considered:
$s \models E \pi$ iff there exists a **fair** path from $s$ satisfying $\pi$;
$s \models A \pi$ iff all **fair** paths from $s$ satisfy $\pi$.

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**Model checking:**

$$[\text{EG } \phi] = \nu Z.[\phi] \cap \bigcap_{fc \in FC} \text{Pre}(\mu Y. (Z \cap fc) \cup ([\phi] \cap \text{Pre}(Y)))$$

where $\text{Pre}(Z)$ is the set of states having a successor in $Z$.


Adding fairness constraints to the card game
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Conclusion and Perspectives
\[ \text{ATLK}_{p_0}^F = \text{FairCTL}, \text{ knowledge and } \text{ATL}_{ir} \text{ with fairness} \]

**Syntax:** CTL \((\text{EX, AG, etc.}), \text{ knowledge (K}_{ag}, \text{ C}_g, \text{ etc.) and strategies (⟨Γ⟩F, [Γ]U, etc.)} \)

**Semantics:** A state \(s\) satisfies \(⟨Γ⟩\ π\) iff there exists a set of memoryless uniform strategies for agents in \(Γ\) such that all fair paths enforced from all \(s’ ∼_Γ s\) satisfy \(π\).
To model check $ATLK_{po}^F$, we defined $ATLK_{fo}^F$ and its model checking

\[ ATLK_{fo}^F = FairCTL + \text{knowledge} + ATL \text{ with fairness} \]

$ATLK_{fo}^F$ semantics: A state $s$ satisfies $\langle \Gamma \rangle \pi$ iff there exists a set of memoryless strategies (not necessarily uniform) for agents in $\Gamma$ such that all fair paths enforced (from $s$ only) satisfy $\pi$. 
To model check $ATLK^F_{po}$, we defined $ATLK^F_{fo}$ and its model checking

$ATLK^F_{fo} = FairCTL + knowledge + ATL$ with fairness

$ATLK^F_{fo}$ semantics: A state $s$ satisfies $\langle \Gamma \rangle \pi$ iff there exists a set of memoryless strategies (not necessarily uniform) for agents in $\Gamma$ such that all fair paths enforced (from $s$ only) satisfy $\pi$.

$ATLK^F_{fo}$ model checking:

$$[[\Gamma]G\phi]^F_{fo} = \nu Z. [[\phi]^F_{fo}] \bigcap_{fc \in FC} Pre[\Gamma](\mu Y.(Z \cap fc) \cup ([\phi]^F_{fo} \cap Pre[\Gamma](Y)))$$
$\text{ATL}_K^{F_{\text{po}}}$ model checking

A state $s$ satisfies $\langle \Gamma \rangle \pi$ iff there exists a set of memoryless uniform strategies for agents in $\Gamma$ which allows $\Gamma$ to enforce $\pi$ in all states indistinguishable from $s$, considering only fair paths.
$ATLK_{po}^F$ model checking

A state $s$ satisfies $\langle \Gamma \rangle \pi$ iff there exists a set of memoryless uniform strategies for agents in $\Gamma$ which allows $\Gamma$ to enforce $\pi$ in all states indistinguishable from $s$, considering only fair paths.

To get all the states satisfying $\langle \Gamma \rangle \pi$:

1. List all the memoryless uniform strategies;
2. Use $ATLK_{fo}^F$ model checking to get states satisfying the property in this strategy;
3. Then restrict to set of undistinguishable states.
ATLK$_{po}^F$ model checking: *Split* algorithm

Split the state/action pairs into memoryless uniform strategies.

1. Get all conflicting equivalence classes;
2. If there are none, the set is itself a memoryless uniform strategy.
3. Otherwise, choose a conflicting equivalence class;
4. Split it;
5. and recursively call *Split* on the rest.
$\text{ATLK}^F_{po}$ model checking example: $\langle \text{player} \rangle F \text{ win}$
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\( ATLK_{po}^F \) model checking example: \( \langle \text{player} \rangle F \text{ win} \)
Improving the algorithm:
alternating between filtering states and splitting strategies

We can alternate between filtering states that belong to a strategy, and splitting non-uniform strategies into uniform ones.

The filtering is correct since $s \not\models_{fo} F \langle \Gamma \rangle \pi \implies s \not\models_{po} F \langle \Gamma \rangle \pi$.

1. Filter current sub-graph for getting states with a strategy;
2. Split on one conflicting equivalence class (if any; otherwise, stop);
3. call the algorithm again with each split sub-graph.
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Conclusion

$ATLK^F_{po}$: branching time, knowledge and strategies under partial observability and (unconditional state-based) fairness constraints.

(Symbolic) model checking algorithm based on $ATLK^F_{fo}$ model checking and splitting the graph into memoryless uniform strategies.
Future work

Develop counter-examples for $ATLK^F_{po}$
(for model understanding, controller synthesis)

Implement a model checker for $ATLK^F_{po}$
with counter-examples generation
(with PyNuSMV, a new Python framework based on NuSMV [5])

Thank you.
Questions?